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AUTHOR Whitman, Nancy C.; And Others
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ABSTRACT

This document provides a model for assessing a school's mathematics program and planning for program improvement. A systematic process for instructional improvement focuses upon students' needs and the identification of successful instructional strategies to meet these needs. The improvement plan and the implementation of intervention strategies are based on the analysis of assessment data. The climate, leadership, and collaborative efforts among people involved are considered as crucial factors; also essential is a common body of knowledge, skills, and attitudes from which to approach instruction. The eight major sections of the manual focus on (1) goals and objectives; (2) evaluation; (3) Foundation Program Assessment and Improvement System (FPAIS), the official curriculum management system of the Department of Education; (4) elementary and intermediate schedules; (5) high school schedules; (6) trends and issues in mathematics education; (7) problem solving; and (8) reading in mathematics. (MNS)

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# MATHEMATICS:
# PROGRAM ASSESSMENT AND IMPROVEMENT PLANNING MANUAL

OFFICE OF INSTRUCTIONAL SERVICES/GENERAL EDUCATION BRANCH • DEPARTMENT OF EDUCATION • STATE OF HAWAII • RS 82-2891 • JUNE 1982

**The Honorable George R. Ariyoshi**
**Governor, State of Hawaii**

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## FOREWORD

As we strive towards the goal of providing quality education in our schools, attention should be focused on assessment and improvement strategies and the development of school-level leaders to implement improvement activities

This document provides a model for assessing a school's mathematics program and planning for program improvement. Staff development is necessary to provide school-level leaders with the skills, knowledge, and strategies for assessment and improvement.

The collaborative efforts of school personnel in the improvement planning process will lead to upgrading mathematics instruction in each classroom and increased achievement of the Foundation Program Objectives.

Donnis H. Thompson

Dr. Donnis H. Thompson
Superintendent of Education

ACKNOWLEDGMENT

Mathematics: Program Assessment and Improvement Planning Manual reflects the efforts of many teachers, principals, district educational specialists and university staff who assisted in the development of the original draft and revisions thereafter. We gratefully acknowledge those who participated in the development and pilot of this Manual.

Special recognition is extended to Nancy C. Whitman and her staff at the University of Hawaii Curriculum Research and Development Group for their work on this Manual.

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# 1. INTRODUCTION

Program Improvement is based on a major assumption that systematic improvement entails clarifying and defining goals, assessing the level of student achievement toward these goals, analyzing the assessment data to determine needs and implications, planning for change, and implementing and evaluating the improvement plan.

This manual represents the first attempt to provide direction and assistance to schools in their efforts to systematically improve instruction and increase student achievement of the Foundation Program Objectives through mathematics. This systematic process for instructional improvement focuses upon students' needs and the identification of successful instructional strategies to meet these needs. The improvement plan and the implementation of intervention strategies are based on the analysis of assessment data. In addition, the climate, leadership, and collaborative efforts among people involved are considered to be crucial factors in program improvement. Also essential is a common body of knowledge, skills, and attitudes from which to approach instruction. This manual incorporates some of the change strategies, content utilization, and the instructional process described in the Foundation Program for the Public Schools of Hawaii, Administrator's Handbook, Office of Instructional Services, RS 80-9253, August 1980.

Each of the major sections of this manual is briefly described below

Section 1. Goals and Objectives

The relationship of the goals and objectives of the mathematics program with the Foundation Program Objectives, the related performance expectations and the essential competencies is shown.

Section 2. Evaluation

A discussion of evaluation as an integral part of the instructional process is provided.

Section 3. Foundation Program Assessment and Improvement System (FPAIS)

The official curriculum management system of the Department of Education is presented. Included in this section is an interpretation of FPAIS at the school level as it relates to mathematics education.

Section 4. Elementary and Intermediate Schedules

Schedules A to G are provided along with guidelines as to the purpose and use of each. Also provided is a flowchart depicting the relationship of these schedules and of school-level assessment and improvement activities.

Section 5. High School Schedules

Schedules A tc G are provided along with guidelines as to the purpose and use of each. A flowchart is provided depicting the relationship of these schedules and of school-level assessment and improvement activities.

Section 6. Trends and Issues in Mathematics Education

This section includes a discussion of the trends and issues of mathematics education.

Section 7. Problem Solving

The nature of the problem-solving process is discussed. Strategies and techniques that may be used in a problem-solving approach are included.

Section 8. Reading in Mathematics

Annotated references on teaching reading in the mathematics classroom are provided. Also tools for assessing the reading level of mathematics materials are provided.

## 2. EVALUATION

Evaluation is an integral part of instruction. Its two purposes are to determine effectiveness of instruction as measured by student achievement and to provide information about ways of improving curriculum and instruction.

At the classroom level, evaluation is the process of determining the value or effectiveness of instruction for the purpose of decision making. The teacher may want to gather information which will determine how effective a particular instructional strategy or material was in meeting identified needs.

A teacher can conduct two basic types of evaluation. They are formative and summative. Formative evaluation takes place before and during instruction. It is a continual process concerned with implementation processes and student progress toward the attainment of specified objectives. Thus, formative evaluation provides the teacher with information during the course of instruction for possible mid-course corrections about any part of the instructional process (preliminary assessment, analysis and planning, instructional strategy, instructional delivery) to help assure that instructional objectives will eventually be met.

Summative evaluation takes place at the end of the instructional unit. It is concerned with student achievement and the degree of success of instructional delivery or instructional strategy. It is the process of gathering and using student achievement data for making judgments - judgments that will determine the next steps to be taken.

At the school or program level, evaluation is viewed as an essential component of an assessment/improvement process. The improvement framework presented in this handbook is based on a major assumption that systematic improvement entails the utilization of an assessment/improvement process

which includes: assessing the level of student achievement toward specified goals; analyzing the assessment data to determine needs and implications; planning for change; implementing and evaluating planned change procedures.

The Foundation Program Assessment and Improvement System (FPAIS), which is the Department's curriculum management system provides a process for conducting assessments, reviewing and analyzing findings and related data, and using the results to plan, program, implement and evaluate educational programs.

A sample application of this process is presented in the outline below.

School Assessment/Improvement Process

The school assessment/improvement process includes the following steps.

1. Data based assessment.
2. Analysis of assessment data and identification of objectives for specific needs.
3. Development of interventions and strategies which will attend to the specific needs (problem-solving approaches).
4. Development of implementation plans, activities, and timeline.
5. Re-assessment of desired change and beginning of the recycling process (evaluation).

A further breakdown of some of these steps could include:

1. Assessment

   a. Activities

      - Identify student strengths and weaknesses
      - Identify current teaching practices
      - Identify teacher philosophy
      - Compare/contrast philosophy with teaching practices and student problems
      - Concur on implications for improvement

   b. Resources

      - Student performance data
      - Standardized test data
      - Teacher perceptions
      - Non-test survey data

2. Analysis

   a. Analytic Framework

      - Content model
      - Philosophy
      - State/school goals and objectives
      - School program

   b. Activities

      - Compare/contrast assessment data with goals and objectives
      - Relate school program and teaching practices to data
      - Determine gaps (needs)

3. Planning and Delivery

   a. Activities

      - Establish priorities with respect to needs
      - Determine strategies with respect to instructional interventions, in-service training, curriculum outlines.
      - Develop implementation plan and timeline

4. Evaluation

   a. Activities

      - Collect student achievement data
      - Measure attainment of program standards

From: The Foundation Program for the Public Schools of Hawaii, Administrator's Handbook, Office of Instructional Service, RS 80-9253, August, 1980, p. E-1, 21.

FPAIS

## 3. FOUNDATION PROGRAM ASSESSMENT AND IMPROVEMENT SYSTEM (FPAIS)

The Foundation Program Assessment and Improvement System (FPAIS) is the official program monitoring system of the Department of Education. Full descriptions of the system are found in the pamphlets: 1) Foundation Program Assessment and Improvement System, Office of Instructional Services, TAC 77-4297, October 1977; 2) Procedural Handbook for the Improvement Component of the Foundation Program Assessment and Improvement System (FPAIS), Office of Instructional Services, TAC 77-4721, December 1977; and 3) Procedural Handbook for the Assessment Component of the Foundation Program Assessment and Improvement System (FPAIS), Office of Instructional Services, February 1981. Each school should have copies of these pamphlets. What follows is a reprint of school-initiated procedures found on pages 20-23 of the Procedural Handbook for the Improvement Component of the Foundation Program Assessment and Improvement System (FPAIS). In addition, overall implementation plans of FPAIS as it relates to mathematics education at the elementary/intermediate levels and high school level are depicted. These plans are fully described in sections four and five of this manual.

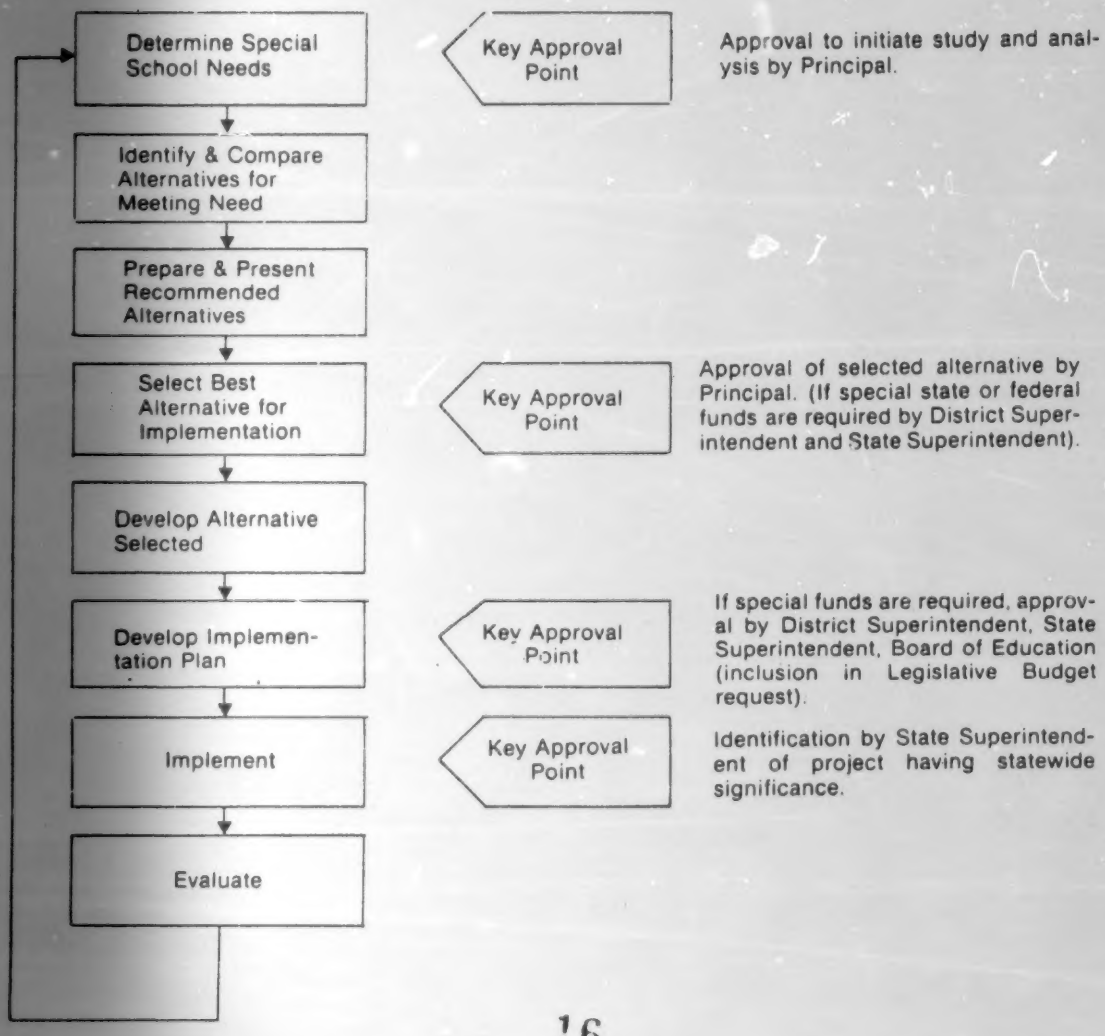
## PROCEDURES FOR SCHOOL-INITIATED CHANGE: SELECTION OR DEVELOPMENT OF SCHOOL IMPROVEMENT PROVISIONS

The criteria for selecting the School-Initiated Change Strategy are the detection and analysis of a high priority curriculum and instructional need at a particular school and the decision that this need can best be met through school initiative and leadership.

At the school level, where dynamic local needs arise within the statewide program, the school may find various assessment and improvement models to be helpful in relating the intention of the Foundation Program Assessment and Improvement System to procedures which are more specific and meaningful for them. An intervention model for assessing and improving the school's reading program is contained in the appendix.

The major activities in the School-Initiated Change Strategy and the key approval points in relation to the activities are contained in the figure which follows.

**Figure 4. School-Initiated Change Strategy**


Determine Special School Needs
Identify & Compare Alternatives for Meeting Need
Prepare & Present Recommended Alternatives
Select Best Alternative for Implementation
Develop Alternative Selected
Develop Implementation Plan
Implement
Evaluate
Key Approval Point
Approval to initiate study and analysis by Principal.
Key Approval Point
Approval of selected alternative by Principal. (If special state or federal funds are required by District Superintendent and State Superintendent).
Key Approval Point
If special funds are required, approval by District Superintendent, State Superintendent, Board of Education (inclusion in Legislative Budget request).
Key Approval Point
Identification by State Superintendent of project having statewide significance.

The section which follows identifies major improvement work phases in relation to the duties and responsibilities of School, District and State personnel.

**Work Phases:** (1) IDENTIFICATION AND ANALYSIS OF PROGRAM NEEDS (2) ANALYSIS, SELECTION AND RECOMMENDATION OF SOLUTION ALTERNATIVE(S)

**SCHOOL**

1. Take initiative in identifying school needs according to Foundation Program Objectives and State policies or guidelines.
2. Organize work group(s) to conduct analysis of problem area(s) and to consider improvement alternatives.
3. Request assistance as appropriate from District/State/external resource agencies.
4. Select best alternative. Forward selected alternative to District Superintendent for review.

**DISTRICT**

1. Provide schools with information regarding new program developments, including general information on a routine basis and specially prepared bodies of information to help particular school improvement efforts.
2. Maintain central collections of curriculum materials provided by State for use by school personnel in improvement studies.
3. Provide consultation services and assist school personnel in identifying and analyzing needs, developing relevant alternatives and selecting alternative for implementation.

**STATE**

1. Provide needed consultation and assistance to District personnel working with schools to identify and analyze needs, develop relevant alternatives and select alternative for implementation.
2. Select, procure, distribute and update collections of curriculum materials maintained in Districts for use by school personnel in improvement studies.

**Work Phase:** (3) DEVELOPMENT OF ALTERNATIVE SELECTED

**SCHOOL**

1. Prepare plans for developing the alternative selected.*
2. Implement plans.
3. Obtain assistance from District/State/external agencies as appropriate.

---

*If developing the alternative selected requires a major investment of time and resources, notify District of intent to undertake the effort as appropriate.

**DISTRICT**

1. Provide all needed assistance to school in developing alternative selected.
2. If State approval is required, provide assistance in preparing subsequent Project Management System (PMS) reports.

**STATE**

1. Provide technical assistance to schools as requested by District.

**Work Phase:** (4) DEVELOPMENT OF IMPLEMENTATION PLAN

**SCHOOL**

1. Develop implementation plan consisting of objectives; installation time frame and schedule; personnel, budget and logistical requirements; support services; evaluation.
2. Involve District as appropriate.
3. Submit implementation plan to District for review.

**DISTRICT**

1. Provide consultation as requested.
2. Review school implementation plan; provide feedback and support.

**STATE**

1. Provide consultation as requested by District.

**Work Phase:** (5) IMPLEMENTATION OF SCHOOL IMPROVEMENT PLAN

**SCHOOL**

1. Coordinate all phases of implementation process.
2. Monitor implementation; make adjustments as necessary.

**DISTRICT**

1. Examine yearly reports on school improvement effort. Respond as needed.
2. Routinely compile information on school improvement efforts in District (number, type, scale). Include aggregated information of District improvement projects and forward to State Superintendent.

**STATE**

1. Routinely compile information on school level improvement efforts statewide, providing periodic summary reports to appropriate authorities.
2. Disseminate information on new program developments statewide as appropriate.

**Work Phase:** (6) EVALUATION OF SCHOOL IMPROVEMENT PLAN

**SCHOOL**

1. Evaluate improvement effort.
2. Provide evaluation report to District.

**DISTRICT**

1. Provide evaluation assistance as appropriate to school.
2. Review evaluation report and provide feedback to school.

**STATE**

1. Provide evaluation assistance as requested by District.
2. Review and respond to evaluation reports as needed.

ELEMENTARY AND INTERMEDIATE SCHOOL
FLOWCHART FOR ASSESSMENT AND IMPROVEMENT ACTIVITIES

(Leadership team directs and oversees Mathematics Committee in all aspects of assessment, planning for improvement, and implementation)

Team completes Schedule A: The School

↓

Committee meets regularly with teachers and begins implementation of assessment plan

↓

Individual teachers complete Schedule B: Grade-Level Assessment

↓

Committee compiles data onto Schedule C: General Summary

↓

Committee identifies and prioritizes needs on Schedule D: Summary of Needs

↓

Committee reviews mathematics programs beginning with those in Schedule F: Mathematics Programs

↓

Committee drafts improvement plan and timeline on Schedule E: Planning for Improvement

↓

Team/committee discusses improvement plan and timeline with teachers and makes necessary revisions/adjustments

↓

School begins implementation of improvement plan using Schedule G
Team coordinates and monitors the implementation

↓

Committee reassesses needs, evaluates progress and revises improvement plan accordingly

↓

Team/committee reviews and updates Schedules A, B, and C

ASSESSMENT
IMPROVEMENT
REASSESSMENT

Goals & Objectives
(of Hawaii State Department of Education)

Evaluation

Schedules A, B, and C

FPAIS Model and Interpretation
Trends and Issues
Problem Solving
Reading in Mathematics
Schedule D

Reviewing of Mathematics Programs (Schedule F)

Schedules E and G

3-6

# School-Level Assessment and Improvement Model for High School Mathematics

HIGH SCHOOL
FLOWCHART FOR ASSESSMENT AND IMPROVEMENT ACTIVITIES

Leadership Team organizes available data related to school and department on Schedule A: School and Department Data, Schedule B: Testing and Student Evaluation and Schedule C: Post High School Information

↓

Team analyzes problem areas reflected in completed Schedules and other sources to complete first draft of summary (Schedule D: Summary)

↓

Team leads mathematics department in discussion, analysis, revision of Schedules A, B, C and first draft of Schedule D

↓

Team/mathematics department completes Schedule D based on this input

↓

→ Team/mathematics department identifies and prioritizes needs on Schedule E: Planning for Improvement
Team begins first draft of improvement plan and timelines on Schedule E
Team utilizes appropriate sections of Schedule F: Materials Assessment

↓

Team leads department meeting to analyze and revise improvement plan and timelines

↓

Team/mathematics department completes improvement plan and begins implementation using Schedule G

↓

Team coordinates and monitors department implementation of improvement plan

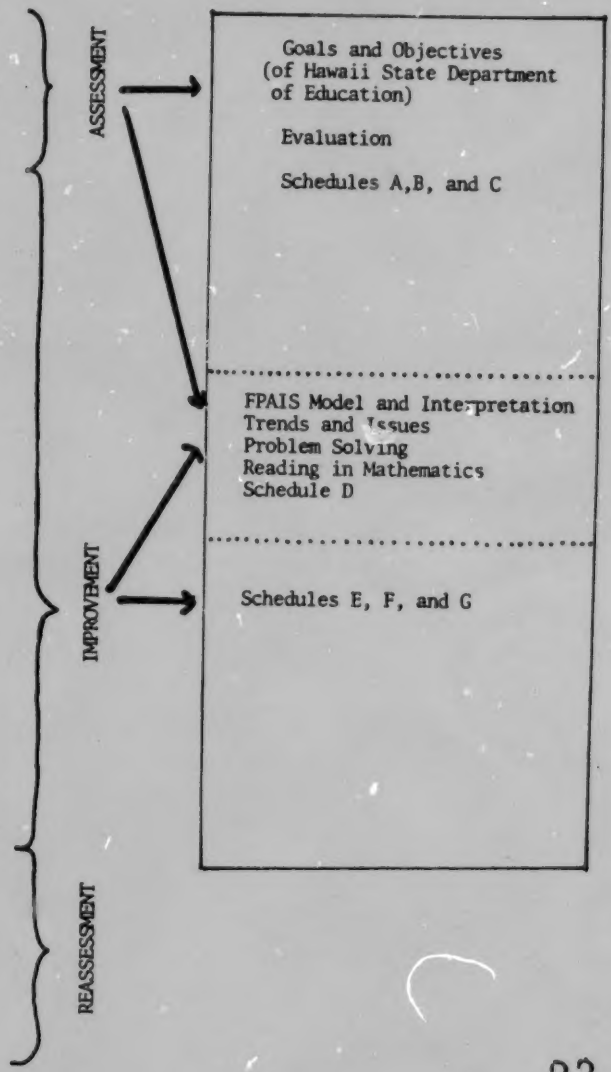
↓

Team/mathematics department reassesses needs and evaluates improvement plan (including adjustments, revisions, etc.)

↓

Team/mathematics department reviews and updates Schedules A, B, C, and D

3-7



# Elementary and Intermediate School Schedules A Through G

## 4. ELEMENTARY AND INTERMEDIATE SCHOOL SCHEDULES

### Guidelines for Using Elementary and Intermediate School Schedules A Through G

### Introduction

The Flowchart for Assessment and Improvement Activities found on the next page represents an interpretation of the Foundation Program Assessment and Improvement System (FPAIS) for school-initiated change in mathematics. The flowchart relates Schedules A through G to the assessment activities, the planning for improvement, and the implementation of the improvement plan.

The model assumes that the leadership team consists of an administrator and at least two teachers. Leadership teams receive support from the District Educational Specialist.

A number of different organizational structures are effective in developing dialogue among faculty to ensure their active involvement in the assessment and improvement activities. These guidelines, however, are written for schools with a somewhat formal structure in which:

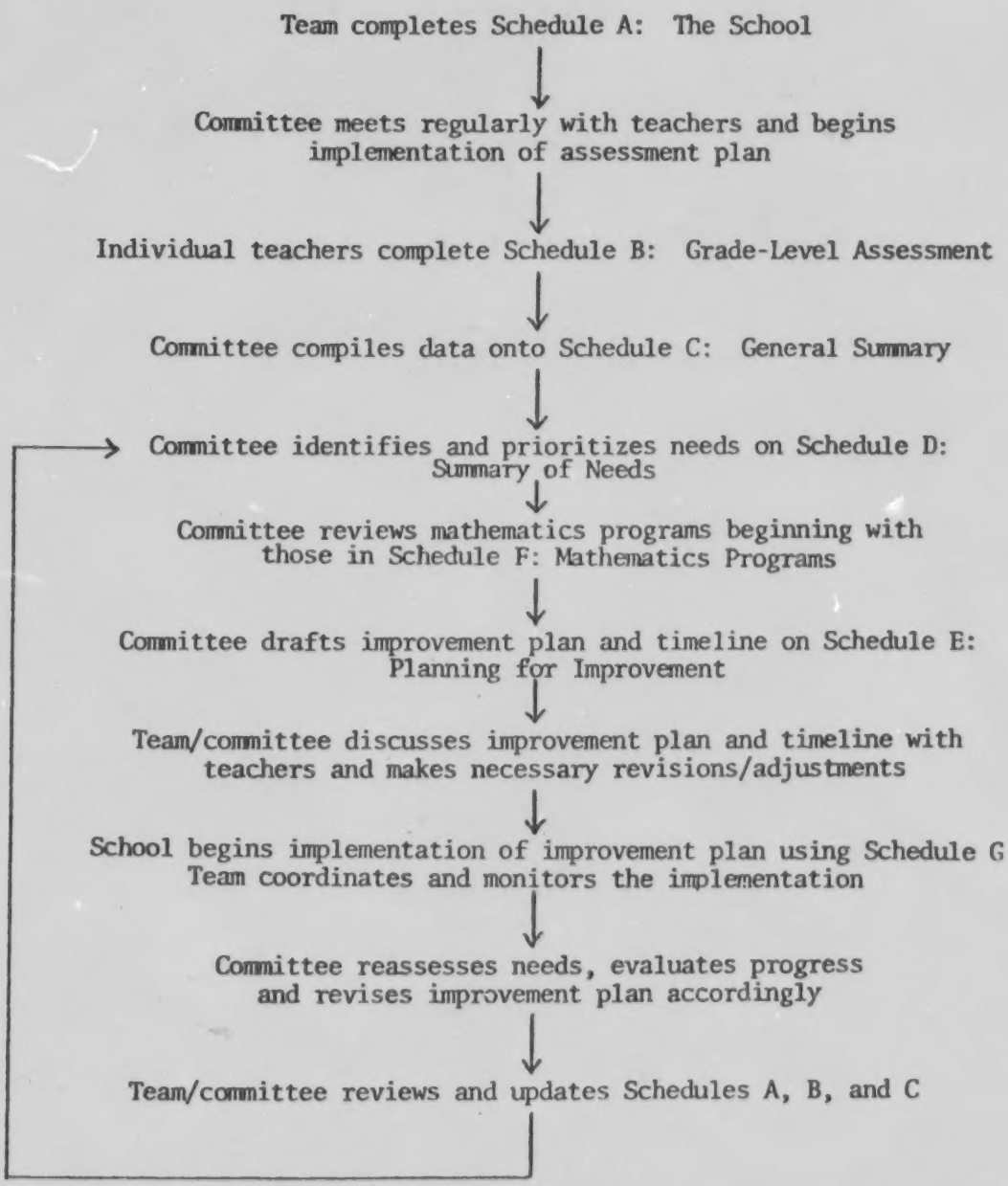
1. At least one teacher on the leadership team is also a member of the school's mathematics committee.
2. The school mathematics committee consists of at least one teacher from each grade level.
3. The mathematics committee member leads the grade-level meetings.

The guidelines offer suggestions to the leadership team directing the school-level mathematics assessment and improvement activities. These guidelines should be interpreted as recommendations based on the experiences of schools that were involved in piloting this model.

Schedules A, B, and C help to organize the assessment activities. These schedules compile the available data relative to curriculum, budget, and so on. The information on these schedules tend to validate people's

ELEMENTARY AND INTERMEDIATE SCHOOL
FLOWCHART FOR ASSESSMENT AND IMPROVEMENT ACTIVITIES

(Leadership team directs and oversees
Mathematics Committee in all aspects of assessment,
planning for improvement, and implementation)


Team completes Schedule A: The School
Committee meets regularly with teachers and begins implementation of assessment plan
Individual teachers complete Schedule B: Grade-Level Assessment
Committee compiles data onto Schedule C: General Summary
Committee identifies and prioritizes needs on Schedule D: Summary of Needs
Committee reviews mathematics programs beginning with those in Schedule F: Mathematics Programs
Committee drafts improvement plan and timeline on Schedule E: Planning for Improvement
Team/committee discusses improvement plan and timeline with teachers and makes necessary revisions/adjustments
School begins implementation of improvement plan using Schedule G
Team coordinates and monitors the implementation
Committee reassesses needs, evaluates progress and revises improvement plan accordingly
Team/committee reviews and updates Schedules A, B, and C

impressions of the school and its mathematics program. The school profile provided by these schedules serves as a basis for school-level planning for improvement. For example, the budget data on Schedule A may be used in determining what areas could be cut back in order to fund the intervention identified to improve the school's mathematics curriculum.

Schedules D, E, and F are related to the school's planning for improvement. This process begins with the leadership team identifying and prioritizing the needs determined from an analysis of the information on Schedules A and C. The leadership team also reviews the trends and issues section of the Manual and mathematics programs such as those described in Schedule F in order to suggest methods of intervention. Following input from the faculty, a three-year improvement plan is finalized on Schedule E.

It is suggested that schools complete the improvement plan within one school year. Moreover, it is strongly recommended that the team begin implementation of the plan to the point where they will be able to continue the momentum the next year. For instance, the team could request for examination copies of materials from their district educational specialists before the end of the first school year in preparation for the next. Other suggestions are provided in Schedule G.

Schedule A The School

The intent of Schedule A is to provide a profile of the school: its organization, student population, professional staff, administrative policies and procedures, and teacher awareness of the Department of Education's goals and priorities for mathematics education.

This information serves as a basis for analyzing data on subsequent schedules and for formulating the school's improvement plan. For example, data on the socio-economic and ethnic background of students are used in

the analysis of standardized test results on Schedule B when comparing the school's population with the population that was used to determine norms such as stanines and percentiles for the standardized test. The background information for mathematics teachers may provide clues to the types of in-service support needed.

The leadership team completes Schedule A. The completed schedule provides a means of sharing relevant information and initiating teacher involvement in the assessment and improvement activities.

The principal has access to most of the data at the school. Data that are not available at the school may be obtained from the district or other Department of Education offices.

Schedule B Grade-Level Assessment

Schedule B is designed to assess the grade-level mathematics program relative to content coverage, problem-solving approaches, and test results. The completion of Schedule B requires a three-step process that involves coordinating the efforts of the leadership team, the mathematics committee, and the individual teachers by grade level.

The completion process serves as a means for dialogue and articulation within and between grade levels. Determining the rating system to be used for Schedule B, for instance, requires school-wide articulation. In order to compile the grade-level assessments into a meaningful school summary on Schedule C, there must be consistency in interpreting the rating symbols within each school.

The three-step completion process described on the next page is based on the school organization described in the introduction to these guidelines. Schools that are organized differently may adapt the process to their needs.

Step 1: Content Coverage

Individual members of the school's mathematics committee lead grade-level meetings to explain the intent and use of information included on Schedule B, emphasizing the importance of school-wide consistency in interpreting the rating symbols selected.

In the first step each teacher is asked to complete the first three columns of the appropriate matrix from Schedule B. Generally a three point system using +, ✓, - or E (excellent), S (satisfactory), N (needs improvement) provides sufficient information on instructional emphases. The intent is to get a picture of the emphases and omissions in the school's mathematics program.

Features of the format that should be explained at the grade-level meetings include:

a) For each grade level the learner objectives from the Mathematics Program Guide are listed in abbreviated form.

b) As indicated in the footnote, the numbers in parentheses following the objectives or clusters of objectives correspond to the numbers on Schedule C Summary Sheet: Content Coverage. This numbering is to facilitate the completion of Schedule C which combines the data for each grade level onto a single school-wide summary.

c) The numbering described above generally reflects the order in which the objectives are introduced in the Mathematics Program Guide. For example, under the topic of whole

numbers for kindergarten are learner objectives for comparison (1), number name and place value (2), and addition and subtraction (3). In grade one another strand, counting (4), is added.

If the school is large, individual committee members should compile this data on a new copy of Schedule B for each grade level. One method is to indicate the number of teachers covering each learner objective. This will facilitate compiling the grade-level data to obtain a school summary on Schedule C.

Step 2: Problem-Solving Approaches

Leadership team (or individual committee members) explains meanings of the six approaches indicated on Schedule B to teachers at a meeting (or by grade level). These approaches are abstracted from the procedural goals described in the Mathematics Program Guide on pages 14-15. A discussion of these approaches is found in the Problem Solving section of this Manual. It is strongly recommended that examples of each of these procedures be demonstrated for teachers. The leadership team and/or the mathematics committee will require prior training by district or state personnel for this step.

Each teacher then completes the Problem-Solving Approaches of Schedule B. Notice that the assessment of approaches used is made relative to a cluster of objectives. The intent here is to indicate those instructional strategies used in addressing the objectives for the particular cluster. Generally, a check to indicate those strategies used provides a profile of those that are emphasized and those that are not.

If the school is large, individual committee members should compile this data onto the grade-level copy of Schedule B begun in Step 1.

Step 3: Test Results

For those grade levels with test data, the test results column of Schedule B is completed at a grade-level meeting. The name of the test (e.g., SAT, CAT, CBM) is indicated in the blank provided at the top of the column. Tests that are administered at more than one grade level provide a basis for comparing growth over the years.

Most leadership teams will need training in analyzing standardized test results. The test results may be rated by grade levels using +, ✓, - (+ for excellent achievement, ✓ for satisfactory achievement, - for needs improvement). The content clusters for which test data are not available are left blank. School-wide consistency in interpreting the rating symbols is also important in this step.

For each grade level that completes all three steps, teachers may begin analyzing the data compiled onto the grade-level copy of Schedule B. Is emphasis on a content area reflected in achievement on test results? If not, what problem-solving approaches were utilized in teaching this content? Could students be provided other types of learning experiences to improve achievement?

Schedule B may also be used to analyze instructional materials relative to coverage of the Mathematics Program Guide Objectives. Teams may find this schedule useful in reviewing the content coverage of new materials or programs in mathematics that the school is interested in adopting.

Schedule C Summary Sheet

Schedule C consists of two parts, Content Coverage and Problem-Solving Approaches. This reflects the concern that the school's mathematics program addresses both mathematical content and mathematical processes.

One part, Summary Sheet: Content Coverage, summarizes the content covered by grade levels and the test results that are available. The other, Summary Sheet: Problem-Solving Approaches, summarizes the approaches (instructional strategies) employed in teaching the various content areas. Both parts together provide a school-wide profile of the mathematics curriculum.

The mathematics committee compiles the grade-level summaries onto the appropriate sections of Schedule C. For most schools, using a check to indicate satisfactory coverage provides sufficient information for the committee to identify areas related to content and procedures in need of improvement.

1. Summary Sheet: Content Coverage

   This part of Schedule C focuses on how well the school's overall mathematics program reflects the content specified in the Mathematics Program Guide by grade level.

   Recall that the numbers in parentheses on Schedule B correspond to the numbers on this part of Schedule C. The descriptions reflect the headings in the Mathematics Program Guide. Generally, the numbering reflects the order in which the clusters are introduced in the Mathematics Program Guide.

   The grade-level assessment of test results can be transferred onto this form. An indication of the type of test and the grade level in the space provided will be helpful in analyzing this summary for

areas in need of improvement.

Considerations involved in the analysis include the following:

a) The summary provides a total school profile. Specific data are available on Schedule B if required.

b) There should be a left-to-right flow of coverage within each area since the numbering reflects the grade-level sequence in which the clusters are introduced in the Mathematics Program Guide.

c) The summary is designed to reflect gaps in content coverage.

d) Emphasis in content coverage should be compared to student achievement on test results. For example, is there an area that is emphasized by teachers which has poor student performance on test results? What implications does this have for instructional strategies, materials, or time-on-task?

2. Summary Sheet: Problem-Solving Approaches

This part of Schedule C focuses on the approaches and strategies used in mathematics instruction. The approaches listed reflect the emphasis on both the state and national levels for the need to incorporate problem solving and language development into the teaching and learning of mathematics.

The completed form reflects those strategies which are being emphasized and which are being neglected. Those strategies being neglected may indicate a need area.

The two parts of Schedule C provide a school profile of the emphases

in content and processes of the mathematics curriculum and serve as a basis for identifying areas in need of improvement. The analysis could include considerations such as which instructional strategies appear to be effective and which may be ineffective for a given content area.

Schedule D Summary of Needs

Schedule D organizes and summarizes the analysis of the school's mathematics program. This process involves identifying areas in need of improvement and prioritizing these needs.

The analysis is conducted by the leadership team and/or the mathematics committee based on the school-wide profile reflected primarily on Schedules A and C. To identify weaknesses in the school's mathematics program, those conducting the analysis need to be familiar with the goals and objectives of the Mathematics Program Guide, as well as the information in the Manual on trends and issues, problem solving, reading in mathematics, and Schedule F: Mathematics Programs.

Weaknesses are identified for areas assessed in the earlier schedules in Parts I and II of Schedule D and a statement of needs is formulated. The needs are then ranked in priority order by the team and/or committee.

The completed Schedule D should be shared with the various faculty groups: mathematics committee, grade levels, or general faculty. Some schools may decide to complete the first draft of the improvement plan on Schedule E before planning a meeting with the faculty group. This decision depends upon the extent of faculty involvement and dialogue in the school's mathematics improvement effort. It is important that the entire faculty have input in the process of finalizing the school's mathematics improvement plan. This process provides the opportunity

for interaction and articulation to develop support for implementation of the identified improvement activities.

Schedule E Planning for Improvement

Schedule E provides a format for developing the school's three-year improvement plan. Each need statement is analyzed relative to considerations such as resources required and constraints imposed. A time-frame is established for addressing needs and implementing the improvement plan.

The leadership team and/or the mathematics committee may choose to review the information in the Mathematics Program Guide and the school's needs. The school may also request assistance from District, State, University, and other resource personnel during their planning.

The draft of the improvement plan including the time frame suggested on Schedule E is shared at a meeting for reaction and input from the faculty. The improvement plan is revised and finalized based on this input. Faculty support and involvement is necessary in the implementation of the school's improvement plan.

An example of a school improvement plan is provided as an appendix to Schedule E. This improvement plan is based on the summary of needs included in the appendix to Schedule D.

It is strongly recommended that the leadership team begin the first steps toward implementation of the improvement plan without a lapse in time. Generally, the team and/or committee has organized into a cohesive working group and the faculty has been involved in the assessment and improvement activities. This situation facilitates the implementation of the improvement plan. Also, a retracing of steps that may be required

after a summer loses the momentum that had been established. A few suggestions for beginning the implementation are provided in Schedule G.

Schedule F Mathematics Programs

Schedule F contains brief desciptions of instructional programs for grades K-8. The programs included do not represent a complete listing of available programs, but represent the range of options that are available.

One purpose of these descriptions is to create an awareness among educators about some of the programs that are available. Some of the learning experiences cited in these descriptions may be new to mathematics teachers or different from those presently in use. These ideas on teaching strategies and student experiences provide members of the leadership team and/or the mathematics committee with further background in their analysis of the school's mathematics program on Schedule D. Some of the programs may be identified as part of the school's improvement plan on Schedule E.

For schools that are considering the adoption of a new mathematics program, these descriptions provide a beginning from which materials can be selected for more thorough study. Copies of materials that a school selects for careful examination and comparison can generally be obtained from the publisher. The District Educational Specialist may also have materials for schools to examine.

Schedule G Suggestions for Implementation

Schedule G provides a few suggestions that may be appropriate for the first few steps in implementing the school's improvement plan.

These initial steps are intended to help "get the ball rolling."

The suggestions do not cover all possibilities, nor are all suggestions appropriate for every school. Rather, several sources that schools may contact for help in implementing their plan and examples of intervention within the school are provided.

ELEMENTARY AND INTERMEDIATE SCHOOL
SCHEDULE A
THE SCHOOL

I. School Data

A. Student enrollment (School Year 19__ to 19__): _____

B. Name of Principal: _____

C. Name(s) of Vice-Principals: _____

D. Number of regular classroom teachers at each grade level:

| | | |
|---|---|---|
| K _____ | 3 _____ | 6 _____ |
| 1 _____ | 4 _____ | 7 _____ |
| 2 _____ | 5 _____ | 8 _____ |

E. Number of other professional personnel:

_____counselors _____resource teachers

_____librarians _____special education teachers

_____other (specify)_____

II. Student Body Data

A. Percent of student body that is:

| | | |
|---|---|---|
| _____ American Indian/Alaskan | Asian or Pacific Islander: | |
| _____ Hispanic | _____ Hawaiian | _____ Korean |
| Non-Hispanic: | _____ Part-Hawaiian | _____ Filipino |
| _____ Black | _____ Chinese | _____ Samoan |
| _____ White | _____ Japanese | _____ Other |

B. Percent of student body for which English is a Second Language: _____

C. Percent of student body from families in the:

_____ Upper Economic Levels

_____ Middle Economic Levels

_____ Lower Economic Levels

D. Percent of student turnover (other than by graduation): _____

E. What information relative to a student's mathematics ability is provided on student transfers?

_____previous math grades _____other

_____test results

How is this information used by mathematics teachers? _____

_____

III. Mathematics Teachers

List all mathematics teachers and complete the table. For the column headed Mathematics or Mathematics Education Background, factors to consider include the teacher's academic major/minor, most recent university work, number of years of teaching, and number of years of teaching mathematics.

| Name | Grade | Is class homo-geneous | Is class hetero-geneous | Is class departmen-talized? | Mathematics or Mathematics Education Background |
|---|---|---|---|---|---|
| | | | | | |
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IV. School Budget

A. Is the school's overall fiscal situation (budget amount and procedures) discussed with the staff? _____

B. Budget Allocation

| School Year | Total School | Total Mathematics Materials | Mathematics Materials Per Student |
|---|---|---|---|
| 19__-__ | $_____ | $_____ | $_____ |
| 19__-__ | $_____ | $_____ | $_____ |
| 19__-__ | $_____ | $_____ | $_____ |
| 19__-__ (projected) | $_____ | $_____ | $_____ |

C. Additional Funding (e.g., Special Needs, Title I, Title IVB, etc.)

| School Year | Total School | Total Mathematics Materials |
|---|---|---|
| 19__-__ | $_____ | $_____ |
| 19__-__ | $_____ | $_____ |
| 19__-__ | $_____ | $_____ |
| 19__-__ (projected) | $_____ | $_____ |

V. School Procedures Relative to Mathematics

A. Budget

1. How is the annual mathematics materials budget determined? _____

_____

2. What criteria is used in allocating this money within the school?

_____

B. Texts and Instructional Materials

1. How are texts and instructional materials selected?

_____ by committee

_____ by administration

_____ by mathematics teacher

_____ by district personnel

_____ other (specify)

2. List all major instructional material resources used by the mathematics teaching staff in your school. This includes textbook as well as non-textbook materials.

| Instructional Material | Grade(s) | Type of Class (e.g., remedial, self-contained, average) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

C. Post Elementary/Intermediate Schools

1. What school(s) do your students feed into? _____

_____

2. What student and curriculum information is provided to the mathematics department of these schools? _____

_____

3. Does your school have anything to do with initial placement of students at their new school? _____ If yes, how is placement determined? _____

_____

D. Meetings

Elementary Schools Only

1. Is there a standing committee in charge of mathematics curricular affairs? _____ If yes, how often does this committee meet? _____ Average length of meeting: _____

2. How often are grade-level meetings held for mathematics? _____ Average length of meeting: _____

Intermediate Schools Only

1. How many times a month are mathematics department meetings held?

   _____ Average length of meeting: _____

2. During the past year, how many department meetings dealt primarily with mathematics curriculum matters? _____

For All Schools

3. Indicate which groups (F = faculty, M = mathematics committee/department, G = grade level) have discussed and studied the school math program in relation to:

   a. the Foundation Program Objectives. _____

   b. the Essential Competencies. _____

   c. the Performance Expectations. _____

   d. the Mathematics Program Guide (including the incorporation of broad goals such as problem solving and communication skills into the curriculum). _____

4. During the past two years, which of the following items, in addition to those given in (3) above, have been discussed in faculty meetings (F), mathematics committee/department meetings (M), and grade level meetings (G)?

   _____ a. Communicating mathematics program concerns to the administration

   _____ b. Communicating mathematics program concerns to the support staff (e.g., special education, counselors)

   _____ c. Mathematics program planning and review

   _____ d. Inservice training

   _____ e. Selection of instructional materials

   _____ f. Mathematics budget

   _____ g. Teacher work assignments

   _____ h. Other (describe)

VI. Testing and Student Evaluation

A. Is there a master list of school test results? _____

B. Who maintains this list? _____

C. Who has access to this list?

_____ district personnel _____ counselors _____ other (describe)
_____ administrators _____ teachers

D. Stanford Achievement Test results (School Year 19__ to 19__)

| Stanines | 1-3 | 4-6 | 7-9 |
|---|---|---|---|
| Grade 2 Mathematics Test | ____% | ____% | ____% |
| Grade 4 Mathematics Test | ____% | ____% | ____% |
| Grade 6 Mathematics Test | ____% | ____% | ____% |
| Grade 8 Mathematics Test | ____% | ____% | ____% |

E. List other tests (including grade level and time of year administered) that are given by your school.

F. Describe how these tests and assessments have been used in:

1. placing students _____
2. assessing mathematics program _____
3. assessing student needs _____
4. other _____

G. Do test results of the past few years indicate an improvement or a decline in the students' knowledge and skills? _____

_____

## VII. Special Student Needs

A. What methods does the school recommend for meeting the needs of students who are:

1. poor or non-readers? _____
2. non-English speaking students? _____
3. recent immigrants? _____
4. mainstreamed handicapped students? _____
5. other slow learners? _____

B. How are the better mathematics students being challenged?

_____ 1. mathematics club

_____ 2. enrichment classes

_____ 3. individual student adaptive programs

_____ 4. acceleration in grades

_____ 5. other (specify) _____

ELEMENTARY AND INTERMEDIATE SCHOOL
SCHEDULE B
KINDERGARTEN

Number of days per week mathematics is taught: _____

Length of period: _____

Reading level of materials for students: _____

Is reading level appropriate? _____

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES | | | | | | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | |
| NUMBERS AND OPERATIONS | | | | | | | | | | |
| • Compares sets (1)* | | | | | | | | | | |
| • Writes numbers 0-9 (2) | | | | | | | | | | |
| • Uses addition and subtraction facts (3) | | | | | | | | | | |
| • Investigates halves (8) | | | | | | | | | | |
| GEOMETRY | | | | | | | | | | |
| • Recognizes geometric figures (18) | | | | | | | | | | |
| MEASUREMENT | | | | | | | | | | |
| • Recognizes relative times<br>• Uses early, late, later, etc. (26) | | | | | | | | | | |
| • Uses money (27) | | | | | | | | | | |
| • Describes temperature (28) | | | | | | | | | | |
| • Compares mass of objects (29) | | | | | | | | | | |
| • Compares lengths<br>• Investigates length through observation (30) | | | | | | | | | | |
| • Orders objects (31) | | | | | | | | | | |
| • Recognizes and develops patterns (32) | | | | | | | | | | |

*Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades K-6.

GRADE 1

Number of days per week mathematics is taught: _____

Length of period: _____

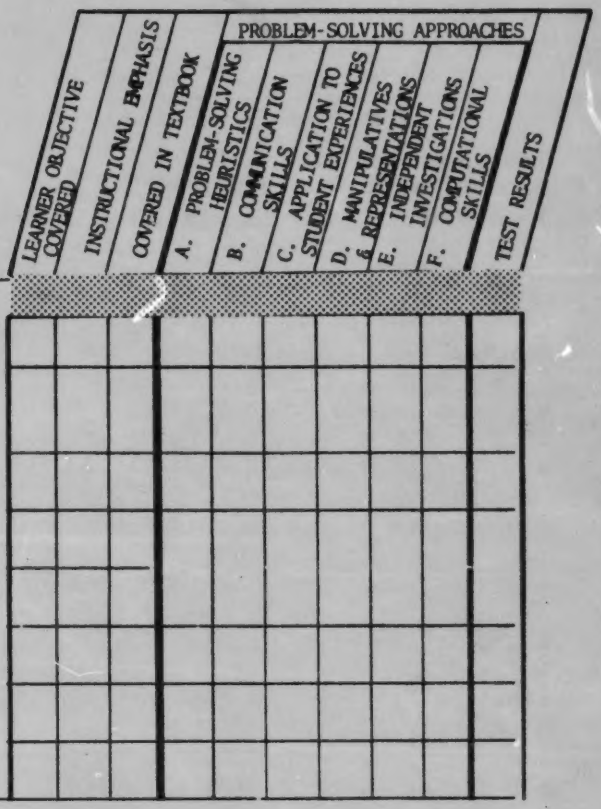
Reading level of materials for students: _____

Is reading level appropriate? _____

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| **NUMBERS AND OPERATIONS** | | | | | | | | | | |
| • Compares numbers (1)* | | | | | | | | | | |
| • Counts by 1's, 2's, 5's, and 10's (4) | | | | | | | | | | |
| • Reads and writes number of tens and ones in 0-99 (2) | | | | | | | | | | |
| • Uses first, second, third, ... tenth (2) | | | | | | | | | | |
| • Relates addition and subtraction | | | | | | | | | | |
| • Uses addition and subtraction facts (3) | | | | | | | | | | |
| • Verifies order and zero properties (3) | | | | | | | | | | |
| • Uses relational thinking (3) | | | | | | | | | | |
| • Adds three numbers (3) | | | | | | | | | | |
| • Recognizes halves, thirds, fourths (8) | | | | | | | | | | |
| **GEOMETRY** | | | | | | | | | | |
| • Geometric figures (maintains and extends) (18) | | | | | | | | | | |
| • Points out similar, congruent, and symmetric objects (19) | | | | | | | | | | |
| **MEASUREMENT** | | | | | | | | | | |
| • Tells time in hours, half-hours (36) | | | | | | | | | | |
| • Understands movement of hands of clock (36) | | | | | | | | | | |
| • Makes change for a quarter (27) | | | | | | | | | | |
| • Describes temperature change on thermometer (28) | | | | | | | | | | |

* Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades K-6.

** An explicit objective of grades K and 3 in the Mathematics Program Guide and an implicit objective of grades 1 and 2.




- Makes change for a quarter (27)
- Describes temperature change on thermometer (28)
- Weighs objects in grams (29)
- Measures length in non-standard units (30)
- Measures length in centimeters
- Compares regions using relative terms (33)
- Compares volumes (34)
- Makes bar graphs (35)

Number of days per week mathematics is taught: _____

Length of period: _____

Reading levels of materials for students: _____

Is reading level appropriate? _____

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
| **NUMBERS AND OPERATIONS** | | | | | | | | | | |
| • Orders numbers to 100 (1)* | | | | | | | | | | |
| • Counts by 3's and 4's (4) | | | | | | | | | | |
| • Reads and writes three-digit numerals | | | | | | | | | | |
| • States number names through "twenty" (2) | | | | | | | | | | |
| • Uses terms eleventh, twelfth, ...twentieth | | | | | | | | | | |
| • Writes different expressions for same number | | | | | | | | | | |
| • Discovers arrangements for even and odd numbers (5) | | | | | | | | | | |
| • Pictures addition and subtraction on number line | | | | | | | | | | |
| • Checks addition and subtraction | | | | | | | | | | |
| • Knows facts through sums of 18 (3) | | | | | | | | | | |
| • Adds three or more numbers | | | | | | | | | | |
| • Groups materials and records results | | | | | | | | | | |
| • Discovers value of ☐ in sentences | | | | | | | | | | |
| • Recognizes zero as identity | | | | | | | | | | |
| • Verbalizes idea of addends, sum, etc. | | | | | | | | | | |
| • Makes smaller groups with same number of objects | | | | | | | | | | |
| • Relates multiplication and addition (5) | | | | | | | | | | |
| • Understands order property for multiplication | | | | | | | | | | |

* Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades K-6.

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| • Uses multiplication facts through products of 25 (6) | | | | | | | | | | |
| • Recognizes halves, thirds, and fourths (8) | | | | | | | | | | |
| GEOMETRY | | | | | | | | | | |
| • *Geometric figures (maintains and extends) (18) | | | | | | | | | | |
| • Sorts plane and solid objects (19) | | | | | | | | | | |
| • Devises ways to determine congruence (19) | | | | | | | | | | |
| MEASUREMENT | | | | | | | | | | |
| • Tells time in hours and minutes | | | | | | | | | | |
| • Names days and months (26) | | | | | | | | | | |
| • Makes time schedules | | | | | | | | | | |
| • Names and writes value of coins | | | | | | | | | | |
| • Uses symbols for cents and dollars | | | | | | | | | | |
| • Reads, interprets, and records temperature (28) | | | | | | | | | | |
| • Weighs objects in kilograms (29) | | | | | | | | | | |
| • Measures length in meters and decimeters (30) | | | | | | | | | | |
| • Uses body parts to determine length | | | | | | | | | | |
| • Compares regions of geometric figures (33) | | | | | | | | | | |
| • Investigates liter and deciliter (34) | | | | | | | | | | |
| • Estimates, measures, and records volume | | | | | | | | | | |
| • Interprets data (35) | | | | | | | | | | |

* An explicit objective of grades 1 and 3 in the Mathematics Program Guide and an implicit objective of grade 2.

GRADE 3

Number of days per week mathematics is taught: _____

Length of period: _____

Reading level of materials for students: _____

Is reading level appropriate? _____

| NUMBERS AND OPERATIONS | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| • Orders numbers to 1,000 (1)* | | | | | | | | | | |
| • Counts by 100's, 10's, 5's, 4's, 3's, 2's, and 1's (4) | | | | | | | | | | |
| • Writes numerals and names place value | | | | | | | | | | |
| • Reads and writes Roman numerals (2) | | | | | | | | | | |
| • Uses ordinal and cardinal numbers | | | | | | | | | | |
| • **Number properties (maintains and extends) (5) | | | | | | | | | | |
| • Adds and subtracts 3-digit numbers | | | | | | | | | | |
| • Adds 3 addends (3) | | | | | | | | | | |
| • Adds and subtracts without paper and pencil | | | | | | | | | | |
| • Uses number line for multiplication and division | | | | | | | | | | |
| • Uses relationship of multiplication and division | | | | | | | | | | |
| • Finds product of three factors (6) | | | | | | | | | | |
| • Relates multiplication to addition and division to subtraction | | | | | | | | | | |
| • Uses facts through products of 81 | | | | | | | | | | |
| • Multiplies with factor less than 100 | | | | | | | | | | |
| • Divides with 1-digit divisor | | | | | | | | | | |
| • Identifies two-thirds, three-fourths | | | | | | | | | | |
| • Reads and writes fractions | | | | | | | | | | |

*Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades K-6.

**An explicit objective of grades 2 and 5 in the Mathematics Program Guide and an implicit objective of grades 3 and 4.

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| • Uses terms numerator and denominator (8) | | | | | | | | | | |
| • Adds and subtracts fractions using pictures | | | | | | | | | | |
| GEOMETRY | | | | | | | | | | |
| • Names and draws geometric figures (18) | | | | | | | | | | |
| • Investigates geometric properties | | | | | | | | | | |
| • Investigates line symmetry (19) | | | | | | | | | | |
| • Draws congruent figures | | | | | | | | | | |
| • Understands geometry terminology (20) | | | | | | | | | | |
| • Locates and forms angles (21) | | | | | | | | | | |
| MEASUREMENT | | | | | | | | | | |
| • Tells and writes time in minutes | | | | | | | | | | |
| • Describes a year in months, days (26) | | | | | | | | | | |
| • Reads and writes money expressions | | | | | | | | | | |
| • Makes change (27) | | | | | | | | | | |
| • Counts and records amount of money | | | | | | | | | | |
| • Describes boiling and freezing points of water on thermometer (28) | | | | | | | | | | |
| • Investigates gram and kilogram (29) | | | | | | | | | | |
| • Finds perimeter (30) | | | | | | | | | | |
| • Explores area (33) | | | | | | | | | | |
| • Measures volume in centiliters and milliliters | | | | | | | | | | |
| • Forms figures and tells volume | | | | | | | | | | |
| • Investigates relationship of capacity and volume units | | | | | | | | | | |
| • Draws graphs (35) | | | | | | | | | | |

Numbers of days per week mathematics is taught: _____

Length of period: _____

Reading level of materials for students: _____

Is reading level appropriate? _____

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
| **NUMBERS AND OPERATIONS** | | | | | | | | | | |
| • Orders numbers to 100,000 (1)* | | | | | | | | | | |
| • Writes numerals to 100,000 | | | | | | | | | | |
| • Reads and writes Roman numerals (2) | | | | | | | | | | |
| • Rounds to nearest thousand (7) | | | | | | | | | | |
| • **Counting--whole numbers (maintains or extends) (4) | | | | | | | | | | |
| • **Number properties--whole numbers (maintains or extends) (5) | | | | | | | | | | |
| • Adds and subtracts whole numbers | | | | | | | | | | |
| • Estimates sums (3) | | | | | | | | | | |
| • Estimates products | | | | | | | | | | |
| • Multiplies with 2-digit multipliers | | | | | | | | | | |
| • Multiplies by multiples of 10 and 100 (6) | | | | | | | | | | |
| • Divides with 1-digit divisor | | | | | | | | | | |
| • Checks multiplication and division | | | | | | | | | | |
| • Reads and writes common and mixed fractions | | | | | | | | | | |
| • Uses mixed numbers | | | | | | | | | | |
| • Orders fractions and mixed numbers (8) | | | | | | | | | | |
| • Finds equivalent fractions | | | | | | | | | | |
| • Adds and subtracts fractions (9) | | | | | | | | | | |
| • Reads and writes decimals (11) | | | | | | | | | | |

* Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades K-6.

** An explicit objective of grade 3 in the Mathematics Program Guide and an implicit objective of grade 4.

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | CONTENT IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| • Adds and subtracts decimals (12) | | | | | | | | | | |
| GEOMETRY | | | | | | | | | | |
| •*Geometric figures (maintains or extends) (18) | | | | | | | | | | |
| •*Terminology (maintains or extends) (20) | | | | | | | | | | |
| •*Angles (maintains or extends) (21) | | | | | | | | | | |
| • Investigates geometric solids (22) | | | | | | | | | | |
| • Identifies figures with symmetry<br>• Recognizes symmetry (19)<br>• Sorts Congruent figures<br>• Sorts similar figures | | | | | | | | | | |
| MEASUREMENT | | | | | | | | | | |
| • Chooses appropriate units of time<br>• Relates different units of time (26)<br>• Compares measurements of time | | | | | | | | | | |
| • Makes change<br>• Adds and subtracts money expressions (27) | | | | | | | | | | |
| • Compares readings on thermometers<br>• Makes observation using Celsius units (28) | | | | | | | | | | |
| • Estimates mass<br>• Demonstrates use of grams, kilograms, and milligrams (29) | | | | | | | | | | |

*An explicit objective of grade 3 in the Mathematics Program Guide and an implicit objective of grade 4.

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES | | | | | | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | |
| • Finds perimeters<br>• Uses addition to find perimeter<br>• Makes guesses about distance | (30) | | | | | | | | | |
| • Finds area using grids | (33) | | | | | | | | | |
| • Investigates relationship of capacity and volume units | (34) | | | | | | | | | |
| • Makes line graph | (35) | | | | | | | | | |

Number of days per week mathematics is taught: _____

Length of period: _____

Reading level of materials for students: _____

Is reading level appropriate? _____

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| **NUMBERS AND OPERATIONS** | | | | | | | | | | |
| • **Comparison-whole numbers (maintains or extends) (1)* | | | | | | | | | | |
| • **Counting-whole numbers (maintains or extends) (4) | | | | | | | | | | |
| • Writes numerals through 1,000,000 (2) | | | | | | | | | | |
| • Reads and writes Roman numerals | | | | | | | | | | |
| • Rounds to nearest hundred-thousand (7) | | | | | | | | | | |
| • Investigates prime and composite numbers | | | | | | | | | | |
| • Investigates multiples and factors (5) | | | | | | | | | | |
| • Determines common multiples and factors | | | | | | | | | | |
| • Searches for common multiples | | | | | | | | | | |
| • Finds products | | | | | | | | | | |
| • Finds averages | | | | | | | | | | |
| • Checks division | | | | | | | | | | |
| • Shows division by 0 is meaningless (3) & (6) | | | | | | | | | | |
| • Divides by 2-digit divisors | | | | | | | | | | |
| • Uses inverse relations of multiplication and division | | | | | | | | | | |
| • Expresses fractions in lowest terms | | | | | | | | | | |
| • Orders fractions | | | | | | | | | | |
| • Compares mixed numbers | | | | | | | | | | |

* Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades K-6.

** An explicit objective of grade 4 in the Mathematics Program Guide and an implicit objective in grade 5.

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| • Expresses fractions as decimals | | | | | | | | | | |
| • *Adds and subtracts fractions (maintains or extends) (9) | | | | | | | | | | |
| • Demonstrates multiplication and division of fractions (10) | | | | | | | | | | |
| • Orders and compares decimals (13) | | | | | | | | | | |
| • Writes place value of digits through ten thousandths (11) | | | | | | | | | | |
| • Rounds decimals (14) | | | | | | | | | | |
| • **Adds and subtracts decimals (maintains or extends) (12) | | | | | | | | | | |
| • Multiplies decimals (15) | | | | | | | | | | |
| • Divides decimals | | | | | | | | | | |
| • Uses ratios for comparisons | | | | | | | | | | |
| • Finds equal ratios (16) | | | | | | | | | | |
| • Devises table to display equal ratios | | | | | | | | | | |
| • Expresses and responds to ratios | | | | | | | | | | |
| GEOMETRY | | | | | | | | | | |
| • ***Geometric figures (maintains or extends) (18) | | | | | | | | | | |
| • **Geometric properties (maintains or extends) (19) | | | | | | | | | | |

* An explicit objective of grades 4 and 6 in the Mathematics Program Guide and an implicit objective of grade 5.
** An explicit objective of grade 4 in the Mathematics Program Guide and an implicit objective of grade 5.
*** An explicit objective of grade 3 in the Mathematics Program Guide and an implicit objective of grade 5.

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES | | | | | | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | |
| • *Terminology (maintains or extends) (20) | | | | | | | | | | |
| • *Angles (maintains or extends) (21) | | | | | | | | | | |
| • Studies cubes and makes copy (22) | | | | | | | | | | |
| • Compares sides, angles, and diagonals of polygons (23)<br>• Sorts polygons | | | | | | | | | | |
| • Locates points on grid (24) | | | | | | | | | | |
| MEASUREMENT | | | | | | | | | | |
| • Computes with measurements of time (26) | | | | | | | | | | |
| • Multiplies and divides money expressions (27) | | | | | | | | | | |
| • Makes observations using C° and F° (28) | | | | | | | | | | |
| • Estimates and verifies weights (29) | | | | | | | | | | |
| • Locates distances or lengths<br>• Measures in millimeters (30) | | | | | | | | | | |
| • Finds area efficiently (33) | | | | | | | | | | |
| • Experiments to find prism volume (34) | | | | | | | | | | |
| • Interprets data in line graph<br>• Finds dimensions from scale drawings (35) | | | | | | | | | | |
| • Measures angles with non-standard units (36) | | | | | | | | | | |

*An explicit objective of grade 3 in the Mathematics Program Guide and an implicit objective of grade 5.

Number of days per week mathematics is taught: _____

Length of period: _____

Reading level of materials for students: _____

Is reading level appropriate? _____

| LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES | | | | | | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|
| | | | A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | |
| | | | | | | | | | |

NUMBERS AND OPERATIONS

- **Comparison--whole numbers (maintains or extends) (1)*
- **Counting--whole numbers (maintains or extends) (4)
- Writes numerals to 100,000,000 (2)
- Reads and writes Roman numerals
- Rounds to nearest million (7)
- **Adds and subtracts whole numbers (maintains or extends) (3)
- Investigates tests for divisibility (5)
- Finds prime factors
- Uses division algorithm (6)
- Estimates quotient
- Determines when 2 fractions are equal (8)
- Expresses fractions as decimal
- Adds and subtracts fractions (9)
- Demonstrates multiplication of fractions (10)
- Gives place value through millionths (11)
- Rounds decimals to hundred-thousandths
- Estimates answers before computing (14)

* Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades K-6.

** An explicit objective of grade 4 in the Mathematics Program Guide and an implicit objective of grades 5 and 6.

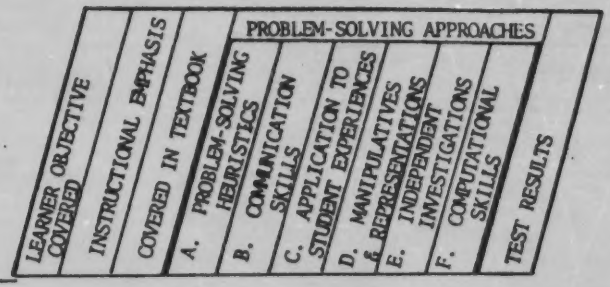
*** An explicit objective of grade 5 in the Mathematics Program Guide and an Implicit objective of grade 6.

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| • Computes with scientific notation (14) | | | | | | | | | | |
| • Expresses fraction as decimal | | | | | | | | | | |
| • **Comparison--Decimals (maintains or extends) (13) | | | | | | | | | | |
| • **Adds and subtracts decimals (maintains or extends) (12) | | | | | | | | | | |
| • **Multiplies and divides decimals (maintains or extends) (15) | | | | | | | | | | |
| • Determines equal ratios (16) | | | | | | | | | | |
| • Understands percent as ratio (17) | | | | | | | | | | |
| GEOMETRY | | | | | | | | | | |
| • ***Geometric figures (maintains or extends) (18) | | | | | | | | | | |
| • ***Geometric properties (maintains or extends) (19) | | | | | | | | | | |
| • ***Terminology (Maintains or extends) (20) | | | | | | | | | | |
| • **Geometric solids (maintains or extends) (22) | | | | | | | | | | |
| • **Coordinates (maintains or extends) (24) | | | | | | | | | | |
| • Classifies polygons (23) | | | | | | | | | | |
| • Classifies and names triangles | | | | | | | | | | |
| • Classifies angles (21) | | | | | | | | | | |

* An explicit objective of grade 5 in the Mathematics Program Guide and an implicit objective of grade 6.

** An explicit objective of grade 3 in the Mathematics Program Guide and an implicit objective of grades 4, 5, and 6.

*** An explicit objective of grade 4 in the Mathematics Program Guide and an implicit objective of grades 5 and 6.




- Uses a compass
- Copies a line segment (25)
- Bisects a line segment

MEASUREMENT

- Recognizes time zones 26)
- Relates money to decimals
- Relates coins to parts of a dollar (27)
- Calculates average cost
- Estimates total cost
- Recognizes variability in temperature (28)
- Relates mass to capacity units (29)
- Investigates fixed perimeter
- Investigates fixed area (30 & 33)
- Determines relationship between circumference and
- Determines surface area of prism (33)
- Finds length, area, volume
- Determines volume efficiently (34)
- Draws line graph (35)
- Estimates and measures angles (36)

Number of days per week mathematics is taught: _____

Length of period: _____

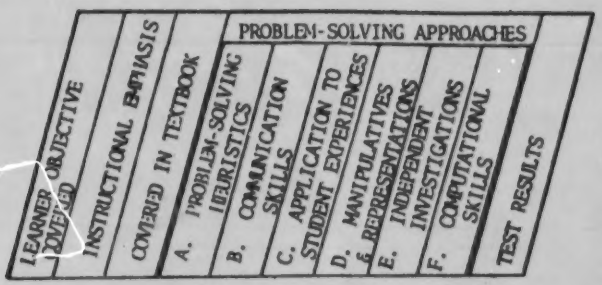
Reading level of materials for students: _____

Is reading level appropriate? _____

| | LEARNER OBJECTIVE COVERED | INSTRUCTIONAL EMPHASIS | COVERED IN TEXTBOOK | PROBLEM-SOLVING APPROACHES: A. PROBLEM-SOLVING HEURISTICS | B. COMMUNICATION SKILLS | C. APPLICATION TO STUDENT EXPERIENCES | D. MANIPULATIVES & REPRESENTATIONS | E. INDEPENDENT INVESTIGATIONS | F. COMPUTATIONAL SKILLS | TEST RESULTS |
|---|---|---|---|---|---|---|---|---|---|---|
| **NUMBERS AND OPERATIONS** | | | | | | | | | | |
| Checks results for reasonableness | | | | | | | | | | |
| Communicates quantitative information | | | | | | | | | | |
| Uses mathematical words and symbols (1)* | | | | | | | | | | |
| Recognizes patterns | | | | | | | | | | |
| Estimates before computing | | | | | | | | | | |
| Investigates systems of numeration | | | | | | | | | | |
| Reads and writes multi-digit numerals | | | | | | | | | | |
| Writes numerals in expanded notation | | | | | | | | | | |
| Explores history of decimal system (2) | | | | | | | | | | |
| Rounds to designated value | | | | | | | | | | |
| Investigates primes and composites | | | | | | | | | | |
| Explores integral exponents | | | | | | | | | | |
| Generalizes properties of whole numbers (3) | | | | | | | | | | |
| Adds and subtracts whole numbers (4) | | | | | | | | | | |
| Multiplies and divides whole numbers (4) | | | | | | | | | | |
| Estimates answers | | | | | | | | | | |
| Investigates least common multiples and common factors (6) | | | | | | | | | | |
| Explores computational short-cuts (6) | | | | | | | | | | |
| Develops speed in mental computation (6) | | | | | | | | | | |
| Investigates mechanical devices for computation (6) | | | | | | | | | | |

* Numbers in parentheses correspond to those in Schedule C Summary Sheet: Content Coverage Grades 7-8.

- Applies order of operations (5)
- Understands relationship of integers and whole numbers (7)
- Adds and subtracts integers
- Multiplies and divides integers (8)
- Orders rational numbers
- Compares fractions and uses symbols
- Names and uses reciprocals
- Recognizes fractions as division
- Adds and subtracts fractions
- Multiplies and divides fractions (10)
- Works with complex fractions
- Understands relationship between fractions and decimals
- Lists advantages of using decimals (11)
- Uses scientific notation
- Estimates square roots
- Adds and subtracts decimals
- Multiplies and divides decimals (12)
- Understands and uses ratios and proportions (13)
- Uses relationship between fractions, decimals, and percents (14)



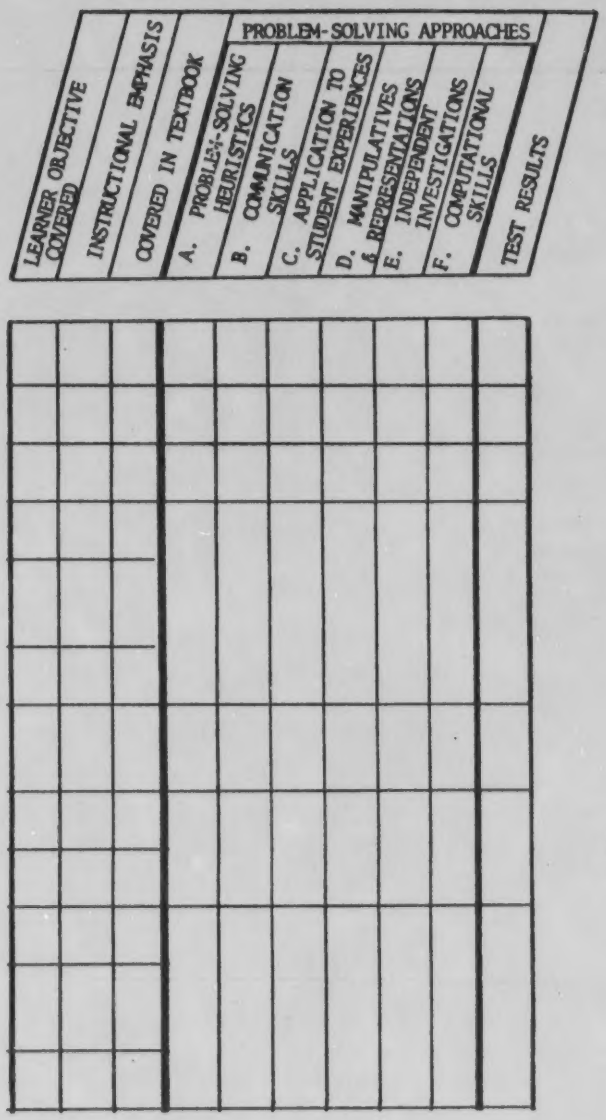

- Investigates how and who uses percents
- Solves percent problems (15)

GEOMETRY

- Draws and explores properties of polygons (16)
- Explores transformations (17)
- Uses proportions in similar figures
- Understands similarity (18)
- Applies properties of geometric figures (19)
- Draws common geometric solids (20)
- Constructs geometric figures
- Explores history of construction (21)
- Uses rectangular coordinate system (22)

MEASUREMENT

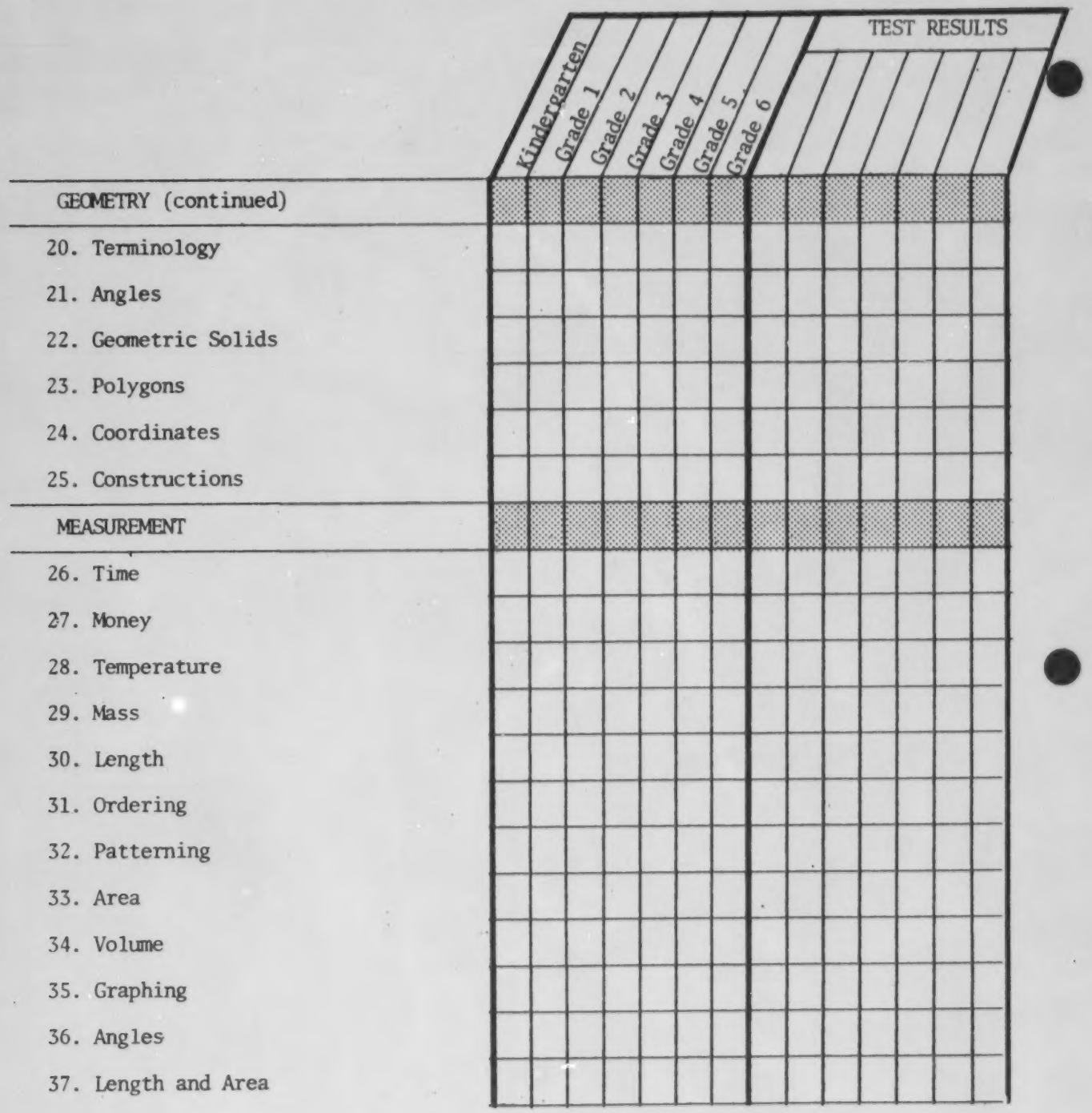
- Uses appropriate tools and units to measure
- Uses measuring tools skillfully (23)
- Estimates measurements (24)
- Uses metric units (25)
- Understands approximate nature of measurement
- Compares U.S. and SI metric units (26)



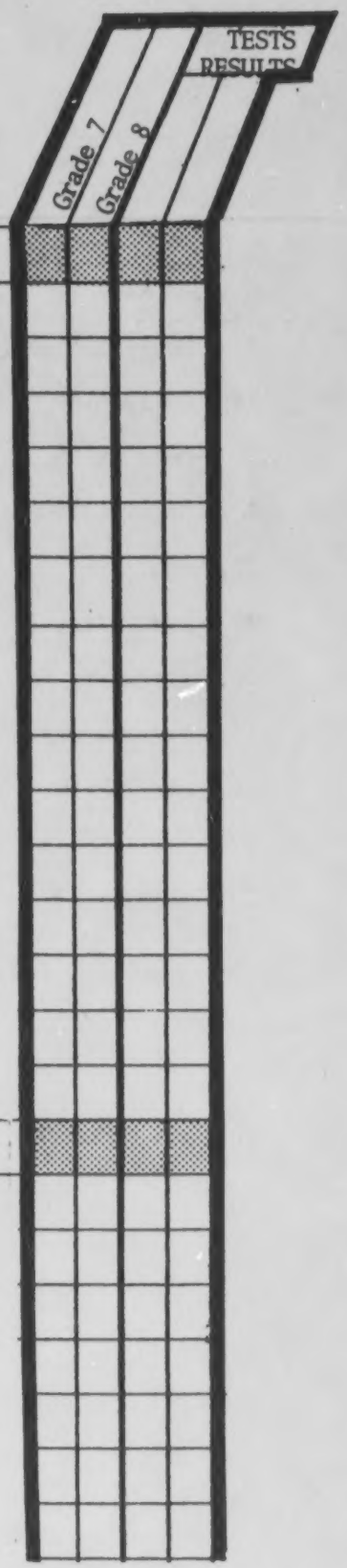

- Estimates and measures angles (27)
- Finds perimeter, area and volume (28)
- Uses formulas (29)
- Finds approximation for pi
- Understands radius, diameter, and circumference (30)
- Uses formula for circumference and area
- Reads, interprets, and makes graphs and tables (31)
- Understands chance
- Understands implications of sampling (32)
- Uses mean, median, mode
- Investigates uses and misuses of statistics (33)
- Conducts statistical studies

SCHEDULE C
SUMMARY SHEET: CONTENT COVERAGE
GRADES K-6

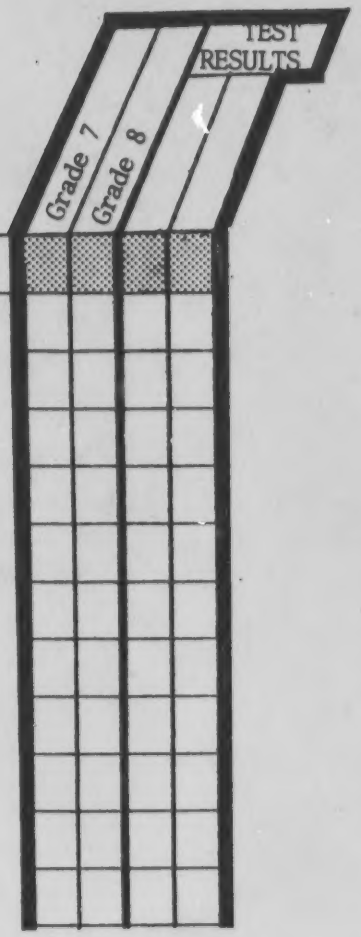
| | Kindergarten | Grade 1 | Grade 2 | Grade 3 | Grade 4 | Grade 5 | Grade 6 | TEST RESULTS | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| WHOLE NUMBERS | | | | | | | | | | | | | | |
| 1. Comparison | | | | | | | | | | | | | | |
| 2. Number Name & Place Value | | | | | | | | | | | | | | |
| 3. Addition & Subtraction | | | | | | | | | | | | | | |
| 4. Counting | | | | | | | | | | | | | | |
| 5. Number Properties | | | | | | | | | | | | | | |
| 6. Multiplication & Division | | | | | | | | | | | | | | |
| 7. Rounding | | | | | | | | | | | | | | |
| FRACTIONS | | | | | | | | | | | | | | |
| 8. Fractions | | | | | | | | | | | | | | |
| 9. Addition & Subtractions | | | | | | | | | | | | | | |
| 10. Multiplication & Division | | | | | | | | | | | | | | |
| DECIMALS | | | | | | | | | | | | | | |
| 11. Names & Place Value | | | | | | | | | | | | | | |
| 12. Addition & Subtraction | | | | | | | | | | | | | | |
| 13. Comparison | | | | | | | | | | | | | | |
| 14. Rounding | | | | | | | | | | | | | | |
| 15. Multiplication & Division | | | | | | | | | | | | | | |
| RATIO | | | | | | | | | | | | | | |
| 16. Ratio | | | | | | | | | | | | | | |
| PERCENT | | | | | | | | | | | | | | |
| 17. Percent | | | | | | | | | | | | | | |
| GEOMETRY | | | | | | | | | | | | | | |
| 18. Geometric Figures | | | | | | | | | | | | | | |
| 19. Geometric Properties | | | | | | | | | | | | | | |


TEST RESULTS
Kindergarten
Grade 1
Grade 2
Grade 3
Grade 4
Grade 5
Grade 6
GEOMETRY (continued)
20. Terminology
21. Angles
22. Geometric Solids
23. Polygons
24. Coordinates
25. Constructions
MEASUREMENT
26. Time
27. Money
28. Temperature
29. Mass
30. Length
31. Ordering
32. Patterning
33. Area
34. Volume
35. Graphing
36. Angles
37. Length and Area

SCHEDULE C
SUMMARY SHEET: CONTENT COVERAGE
GRADES 7-8




| | |
|---|---|
| NUMBERS AND OPERATIONS | |
| 1. Problem-Solving Strategies | |
| 2. Numeration | |
| 3. Whole Number: Concepts | |
| 4. Whole Number: Computation-Basic Algorithms | |
| 5. Whole Number: Computation-Multioperational Problems | |
| 6. Whole Number: Mathematical Applications of Concepts and Computations | |
| 7. Integers: Concept and Relationship to Whole Numbers | |
| 8. Integers: Basic Operations (+, -, x, ÷) | |
| 9. Fractions: Concepts | |
| 10. Fractions: Basic Operations (+, -, x, ÷) | |
| 11. Decimals: Concepts | |
| 12. Decimals: Computation | |
| 13. Ratios and Proportions: Concepts & Application | |
| 14. Percents, Fractions, & Decimals: Relationships | |
| 15. Percents | |
| GEOMETRY | |
| 16. Polygons | |
| 17. Geometric Transformations | |
| 18. Similarity | |
| 19. Geometric Properties | |
| 20. Geometric Solids | |
| 21. Constructions | |
| 22. Coordinates | |

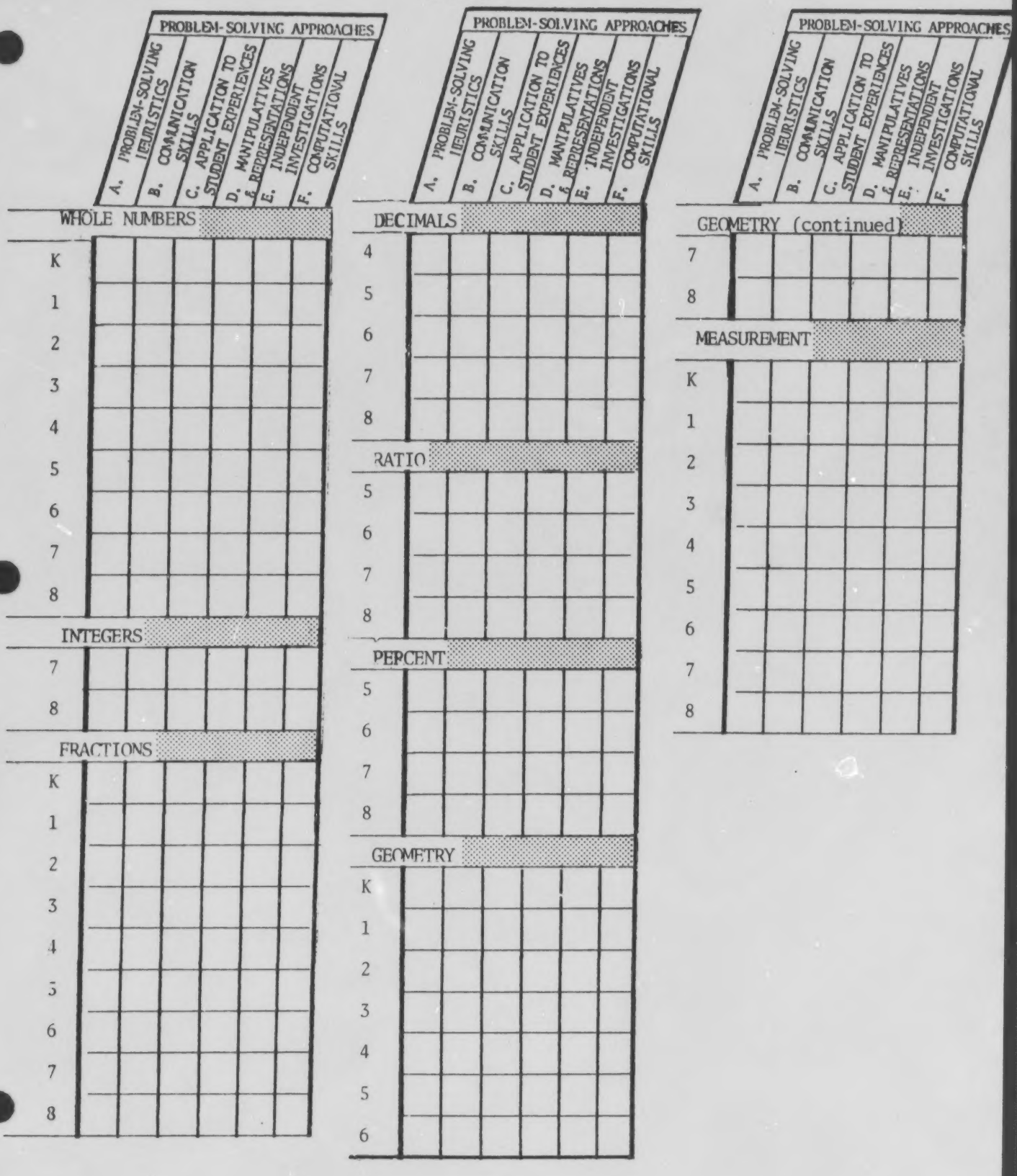



## MEASURMENT

23. Measuring Tools
24. Estimation
25. Metrics and Customary Units
26. Nature of Measurement
27. Angles
28. Perimeter, Area, and Volume
29. Formulas
30. Circle: Relationships
31. Graphs and Tables
32. Probability
33. Statistics

SCHEDULE C

SUMMARY SHEET: PROBLEM-SOLVING APPROACHES
GRADES K-8



ELEMENTARY AND INTERMEDIATE SCHOOL
SCHEDULE D
SUMMARY OF NEEDS

Schedule D should reflect those aspects of the school's mathematics program (e.g., curriculum, staff, instructional materials, and so on) that require intervention.

Use the information on Schedules A, B, and C, the policies of the school, district or state that are related to mathematics education, your own experiences, and relevant information from other sources to complete this schedule.

I. Program Analysis

To complete this section refer to Schedule C. If information for a particular grade level is required, refer to Schedule B.

A. Content

1. Identify content that needs to be deleted.

2. Identify new content that needs to be incorporated into the mathematics program.

B. Problem-Solving Approaches

Identify approaches that need to be incorporated into or emphasized in the mathematics program.

C. Student Evaluation

Identify content areas in which test results do not reflect the emphasis given to them in the classroom.

II. Analysis of Other Areas

List additional aspects of the mathematics program that require intervention relative to:

A. student groups whose needs are not being met in a satisfactory manner. (Refer to part VII of Schedule A.)

B. administrative procedures (e.g., budget, school liaison).

C. curriculum/program concerns (e.g., instructional materials).

D. staff (e.g., inservice needs).

E. other.

# A SAMPLE OF SCHEDULE D

## SCHOOL MATH IMPROVEMENT PROJECT

## SUMMARY OF NEEDS AND MATH IMPROVEMENT PLAN

compiled by Math Leadership Group

## SUMMARY OF NEEDS

### I. Program Analysis

#### A. Content

1. All grade levels use Addison Wesley materials as the basal program and supplement with various other kits, programs, etc.

2. All grade levels have examined the Math Performance Expectations and are providing for them in the grade level program.

3. Content that needs to be modified includes:

| | |
|---|---|
| Grades K-1 | None |
| " 2-3 | None |
| " 4 | De-emphasize fractions for slower students. |
| " 5 | De-emphasize percent and ratio for slower students. |

4. Content that needs to be added or supplemented include:

| | |
|---|---|
| Grades K-1 | None |
| " 2-3 | (Refer to Attachment 1) |
| " 4-5 | For groups using MIOW, add standard or English measures as MIOW uses metric only, Add Roman numerals |

#### B. Approaches (Teaching Strategies) That Need to be Emphasized

1. Problem solving at all grade levels.

2. Independent investigations at all levels beginning at Grade 2.

3. Relating to student's environment beginning at Grade 3.

C. Student Evaluation

1. Grade 2 SAT scores range from stanine 5-7.

2. " 4 " " " " " 4-7.

3. " 6 " " " " " 3-6.

4. Percentage of students scoring in below average range for 1979:

   Grade 2 (18%) Grade 4 (16%) Grade 6 (26%)

5. Lowest stanine scores are in math applications.

6. Content areas in which test results do not reflect the emphasis given in classrooms.

   a. Word problems at all grade levels.

   b. Additionally, at:

      K-1st. . . . . see Attachment 2
      2nd-3rd. . . . . see Attachment 3
      4th-5th. . . . . see Attachment 4

II. Analysis of Other Areas

A. Student Groups Whose Needs are not Being Met Satisfactorily Include:

1. Slowest students needing remedial math in all grades.

2. LES students.

3. Top ability students in all grades.

B. Administrative Procedures

1. Testing--identify need for pre and post tests by grade levels.

2. Math committee--expand role of math committee members to assist leadership group in implementing improvement plan.

C. Curriculum Concerns

1. Instructional materials--policy on the use of supplementary materials with specific groups needs to be clarified.

2. Students completing grade level work--all teachers need to understand and adhere to agreed on approach with such students.

3. Articulation between grade levels--work out understandings between grade levels such that each teacher knows the areas the next grade level would like to have emphasized or mastered etc.

4. Math Guide--arrange inservice training to assure everyone's familiarity with the Mathematics Program Guide.

D. Staff

1. Inservice training needs relate mostly to problem solving and applications, but also include use of manipulatives such as Cuisenaire Kits.

2. Orientation to UH developed materials and CDA Math.

3. Consider using visitations, aides.

III. State of Needs (in priority order)

Priority

1 Articulation of grade level content, ie, what needs to be deleted or de-emphasized and what needs to be emphasized. Also, stipulate policies on supplementary materials and students completing grade level work before end of year.

2 Orientation to and incorporation of the following approaches. (teaching strategies) in instruction:

Problem solving
Independent investigations
Relates to student environment

3 Orientation to the Mathematics Program Guide.

4 Program provision for top group of students.

5 Identification of grade by grade pre and post test needs.

6 Inservice training of staff in relation to procedures, materials, top students.

7 Review need to provide remedial instruction for such students in Grades K-1 and LES category not presently provided for.

ELEMENTARY AND INTERMEDIATE SCHOOL
SCHEDULE E
PLANNING FOR IMPROVEMENT

I. Analysis of Needs

For each of the needs compiled in part III of Schedule D:

A. identify additional information that will be needed for implementation.

B. identify resources that will be required (e.g., materials, services, funding).

C. identify constraints (e.g., time, personnel, funding).

D. estimate the amount of time it will take to complete each component.

E. other considerations.

II. Determination of Timeline

A. Listing of Priorities

1. Refer to the needs analyzed above in Part I. Determine which components of those needs the school will address during each of the next three years.

2. List these in the appropriate space below.

3. Prioritize the needs within each school year.

School Year 19__-19__

School Year 19__-19__

School Year 19__-19__

B. Establishing Time Frame

1. Record priorities for each year on timeline. (Make three copies of this page and use one copy for each year.) Each priority should include a breakdown of all work that must be completed for implementation.

2. Estimate the amount of time necessary to accomplish each of these priorities.

3. Indicate on the timelines when each of those sections will be accomplished during the school year.

| | YEAR:_____ | | | |
|---|---|---|---|---|
| | Qtr 1 | Qtr 2 | Qtr 3 | Qtr 4 |
| Priority 1 | | | | |
| Priority 2 | | | | |
| Priority 3 | | | | |

A SAMPLE OF SCHEDULE E

MATH IMPROVEMENT PLAN

I. Analysis of Needs Prioritized in Summary

A. Articulation of Grade Level Content.

1. Provide grade level meeting time to:

a. Refine or further develop listing of needed deletions and additions.

b. Identify principal areas of emphasis expected at each grade level.

2. Utilize leadership group structure and math committee to complete above task in one semester.

3. Delineate policies on supplementary materials and students completing grade level work before end of year.

B. Incorporation of Teaching Approaches

1. Orient teachers to Math Improvement plan at staff meeting.

2. Follow up with subsequent staff meeting to orient staff to the identified teaching procedures needed. Use members of leadership group or resource person(s). Complete by end of first semester but follow up as necessary.

C. Orientation to Math Program Guide

1. Present overview at staff meeting.

2. Follow with grade level meetings for reviewing specific portions separately by each grade level.

D. Program for Top Students

1. Identify size of group by grade levels.

2. Determine how needs of this group are to be provided for, ie, through a specified set of materials, via projects, etc.

3. Identify materials suitable for this group.

4. Approach tasks above in second or third year of improvement effort, after other preliminary tasks are completed.

E. Identification of Pre and Post Test Needs

1. Identify needs by separate grade levels.

2. Where needed, determine whether standardized or other tests based on grade level program are preferable.

3. Review school developed tests utilized in 1977 assessment for appropriateness for annual use.

4. Decide needs and measures to be utilized by end of first year of improvement cycle.

F. Staff Inservice Training

1. On the basis of identified needs, develop list of available inservice training offerings and resource persons.

2. Arrange for district educational specialist in math to provide some program directions.

3. Schedule some flex-time inservice sessions annually.

G. Remedial Instruction for LES students and K-1 level

1. Determine appropriateness of modifying coverage.

2. Determine need for and if appropriate provide such coverage through Title I program.

II. Timetable of Improvement Activities

Prioritized Activities by School Year and Instructional Quarter

| 1980-81 | Instructional Qtr | | | |
|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 |
| 1. Dissemination and review of improvement plan | * | | | |
| 2. Articulation of grade level content, emphasis, and policies. | * | * | | |
| 3. Orientation to needed instructional procedures | | * | | |
| 4. Initial training on problem solving and applications | | | * | |
| 5. Examination of UH developed curriculum materials | * | | | |
| 6. Review of MPG | * | * | | |
| **1981-82** | **Q1** | **Q2** | **Q3** | **Q4** |
| 1. Determine need for and decide pre-post testing program by grade levels | * | * | | |
| 2. Continuing training activities related to problem solving and other instructional procedures | | * | * | |
| 3. Orientation to CDA Math | | | * | |
| 4. Expanding remedial math coverage | | * | * | |
| **1982-83** | **Q1** | **Q2** | **Q3** | **Q4** |
| 1. Continue training activities related to improvement | | * | * | |
| 2. Determine need for and decide program arrangements for top achieving students | | * | * | |
| 3. Begin assessment of improvement program | * | * | * | |

ELEMENTARY AND INTERMEDIATE SCHOOL
SCHEDULE F
MATHEMATICS PROGRAMS

Schedule F provides information to educators who are considering the adoption of a new mathematics program. A brief description of some of the mathematics programs available for use at the elementary and secondary levels are included for the following:

1. Comprehensive School Mathematics Program, (CSMP), grades K-6
2. Curriculum Development Associates, (CDA), grades K-6
3. Developing Mathematical Processes, (DMP), grades K-6
4. Distar Arithmetic, grades K-6
5. Individualized Mathematics Program, (IMP), grades K-12
6. Intermediate School Mathematics Program, (ISMP), grades 7-8
7. Mathematics Resource Project, grades 6-9
8. Nuffield Mathematics Project, grades K-8
9. School Mathematics Program, (SMP), grades 7-12

All of the above programs, except for the Nuffield Mathematics Project and the Mathematics Resource Project, are complete mathematics programs. That is, the materials are provided for a full year of instruction for each of the specified grade levels. The two partial programs are to supplement existing programs or to fully develop a limited number of specific mathematical concepts.

All of the above programs have some acceptance by mathematics educators as providing an effective curriculum. These programs do not, however,

represent a complete listing of all available programs. These selected programs are meant to be representative of the many different types of programs that are currently available. Each description will contain the following categories:

1. Development of Materials
2. Costs of Materials
3. Inservice Education Required
4. Mathematical Content
5. Experiences for Students
6. Teaching Strategies
7. Summary
8. Bibliography

As stated earlier, the descriptions are brief. They are intended only as a beginning from which materials can be selected for a thorough study and to create an awareness in educators about some of the alternative programs that are currently available.

Checklists are provided to compile summaries which will be valuable later in determining a plan of action for improving the school's mathematics program.

Checklist: Available Programs

| Program | These materials are worth pursuing: as complete program | as partial program | as supplementary materials | as a teacher resource | for other reasons | not at all | Comments |
|---|---|---|---|---|---|---|---|
| 1. Comprehensive School Mathematics Program (CSMP) K-6 | | | | | | | |
| 2. Curriculum Development Associates (CDA) K-6 | | | | | | | |
| 3. Developing Mathematical Processes (DMP) K-6 | | | | | | | |
| 4. Distar Arithmetic K-6 | | | | | | | |
| 5. Individualized Mathematics Program (IMP) K-12 | | | | | | | |
| 6. Intermediate School Mathematics Program (ISMP) 7-8 | | | | | | | |
| 7. Mathematics Resource Project 6-9 | | | | | | | |
| 8. Nuffield Mathematics Project K-8 | | | | | | | |
| 9. School Mathematics Program (SMP) 7-12 | | | | | | | |
| 10. Other | | | | | | | |

Checklist: Textbook Series

| Program | These materials are worth pursuing: as complete program | as partial program | as supplementary materials | as a teacher resource | for other reasons | not at all | Comments |
|---|---|---|---|---|---|---|---|
| 1. Mathematics in Our World (Addison-Wesley) K-8 | | | | | | | |
| 2. Growth in Mathematics (Harcourt, Brace, Jovanovich) K-8 | | | | | | | |
| 3. Harper & Row K-6 | | | | | | | |
| 4. Heath Mathematics K-8 | | | | | | | |
| 5. Holt School Mathematics K-8 | | | | | | | |
| 6. Mathematics (Houghton Mifflin) K-8 | | | | | | | |
| 7. Laidlaw Mathematics K-8 | | | | | | | |
| 8. Macmillan Mathematics K-8 | | | | | | | |
| 9. Real Math (Open Court) K-6 | | | | | | | |
| 10. S.R.A. Mathematics K-8 | | | | | | | |
| 11. Mathematics Around Us (Scott, Foresman) K-8 | | | | | | | |
| 12. Mathematics for Mastery (Silver Burdett) K-8 | | | | | | | |

1. Comprehensive School Mathematics Program (CSMP)

The Comprehensive School Mathematics Program (CSMP) has been approved by the Joint Dissemination and Review Panel of the U.S. Office of Education and the National Institute of Education as an "exemplary" curriculum. The first schools to pilot the program in Hawaii were Pearl Ridge, Waiau and Ala Wai Elementary Schools.

1. Development of Materials

CSMP was prepared by mathematics educators representing all grade levels for CEMREL, Inc., a non-profit organization.

The authors decided to concentrate on "What is the ideal content for school mathematics?" and consider secondarily criteria such as "What can students do?" and "What can teachers teach?" The planning stage for this program was lengthy and changes continue to be made on the basis of field testing. The resulting curriculum covers the content ordinarily found in most present programs. Further, it covers more objectives found in the Mathematics Program Guide (MPG) than most programs presently in use.

2. Description of Materials

The CSMP materials are organized into Kindergarten, first grade, upper primary grades (2 and 3) and intermediate grades (4, 5, and 6). For each grade level teacher materials and student materials are available in classroom sets that accommodate thirty students.

The teacher materials include guides, posters, and kits for class demonstrations. Student materials include worksheet booklets, storybooks and manipulatives. In addition, hand-calculators are needed for some of the lessons.

3. Inservice Education Required

Teachers new to CSMP are required to have special training. This training is conducted by a coordinator who must have attended a summer workshop given by CSMP. A cadre of coordinators has been formed to provide the in-service training for the state. Teachers are involved in an initial training which is ordinarily completed prior to the opening of school. In addition, the coordinator also schedules regular meetings with the teachers to discuss problems and obtain feedback on progress.

Schools that are interested should contact their district educational specialist for mathematics.

4. Mathematical Content

According to a CSMP overview of the program, the curriculum "contains the best of mathematical content essential for understanding the nature of mathematics and its ever increasing applications in the real world" and "...present(s) the content in a humanistic fashion...to render it accessible, exciting and meaningful to young children."

There are four major mathematical strands in CSMP: The World of Numbers; Strings and Arrows; Geometry and Measurement; and Probability and Statistics. Strings are used to collect and classify data relevant to a particular problem while arrows are used to compare and analyze sets and operations on them.

CSMP believes that mathematics is a unified whole and should be treated as such. For that reason the content is developed in a spiral approach. Students build "interlocking experiences of increasing sophistication." It is also designed to extend experiences a child has already encountered. Many of the experiences in CSMP are problem solving oriented and are presented in game-like or story settings.

5. Experiences for Students

Activities classified under the headings Non-Verbal Languages, the Hand Calculator, and Pupil Materials provide important experiences for students.

Three so-called "Non-Verbal Languages" permeate the materials for all grades: the language of strings, the language of arrows, and the Papy Minicomputer. CSMP describes the Minicomputer as "a simple device consisting of a set of colored squares on which checkers are placed....(it) is a type of abacus which combines the usual positional system with the fundamental notion of doubling...."

Calculators are used beginning in grade 1. They are included to remove the burden of mechanical computation so students may concentrate on thinking and to provide instant feedback so children can make and test conjectures. Actually, in CSMP the calculator is a way of enriching the program. The possibilities for use are numerous. Further, CSMP has found that the calculator has a beneficial effect on students' interest and understanding. It is used as a pedagogical rather than a computational tool.

Most of the independent work within the program is done through the use of four kinds of pupil material, described below.

a. Worksheets. Three levels of difficulty are identified and worksheets correspond to specific lessons.

b. Workbooks. These too are for individual work. The work becomes progressively more difficult as the student proceeds through the pages.

c. Storybooks. This collection of 24 stories for young people are independent of each other, require little reading (of words) and need no teacher explanation. They are portrayed by the language of strings and arrows and by the minicomputer.

d. Story-workbooks. These are similar to the storybooks and engage pupils in reading about and doing mathematics in interesting story situations.

6. Teaching Strategies

Carefully prepared Teacher's Guides have been written which provide background information about the program, a layout of day-to-day lessons, and the lessons themselves.

The lessons are very complete and nearly every one is in the form of a sample dialogue with questions the teacher can ask and possible answers by pupils.

Following are a few objectives that teaching strategies are expected to accomplish:

a. Secure a maximum of teacher-student and student-student interaction.

b. Help pupils to understand that mathematics is pictorial and verbal as well as written.

c. Aid pupils to invent ways to do complicated arithmetical problems without recourse to algorithms.

d. Encourage students to apply their curiosity, intuition, and powers of reasoning.

7. Summary

A CSMP publication states, "CSMP has found it difficult to even begin discussing the program except through an in-depth hands-on exposure... usually in the form of a two or three day workshop."

Evaluations by CSMP staff and also by persons not associated with CSMP indicate that CSMP students learn traditional skills at least as well as comparable non-CSMP students and that of the two groups, CSMP students show a higher level of enthusiasm and interest in their mathematics program. The reports also indicate that CSMP students do better in dealing with mathematical processes such as relational thinking, estimation, large numbers, and certain word problems. Listed below are some other generalizations about CSMP. Some are non-evaluative with the reader allowed to decide their worth.

a. There is no format to assess mastery of student skills. Teachers are urged to use any monitoring device with which they feel comfortable.

b. Work with time is omitted. All measurement is done with metric units.

c. There is a strong reliance on games and fantasy and puzzle stories.

d. Few, if any, pages of drill are found. Program authors feel there is a surprising amount of practice throughout the program. Many situations are provided where ability to calculate is needed to respond to a situation and calculation is considered to be a tool to do interesting things rather than a chore to be done for its own sake.

e. There is a strong emphasis on estimation and mental arithmetic.

f. The program has a spiral development; there is repeated exposure

to a topic on increasingly sophisticated levels to produce regular growth.

g. Reading by students is at a minimum, especially in the lower grades.

8. Bibliography


a. CSMP materials are available from CEMREL, Inc., 3120 59th Street, St. Louis, MO 63139. A "List of Available Materials" can be obtained by writing. In addition to teacher and student books, these include evaluation reports, brochures describing various aspects of the program, and about 30 booklets or articles which describe many features of CSMP.

b. Lockard, J. D. (ed.). Twenty Years of Science and Mathematics Curriculum Development, The Tenth Report of the International Clearinghouse. College Park, MD: University of Maryland, 1977. This report contains a short description of CSMP.

c. National Council of Teachers of Mathematics. Mathematics Learning for Early Childhood, Thirty-seventh Yearbook. Reston, VA: NCTM, 1975. A short review of CSMP is included in this yearbook.

d. Woodward, E. "CSMP: A New Aiternative in Elementary School Mathematics." Arithmetic Teacher. February 1980, pp. 20-26. This article is a short description of CSMP which focuses on grades K-4.

## 2. Curriculum Development Associates (CDA) Mathematics

CDA Math is a program in mathematics for grades 1-6. It includes booklets for students, teacher resource books and kits of materials for parents to use with their children. The first schools to pilot the program in Hawaii were Kualapuu and Wilcox Elementary Schools.

### 1. Development of the Materials

CDA Math has been developed over a period of years beginning first as teacher resource materials and culminating in books for students. All materials are published commercially by Curriculum Development Associates, Inc.

This program differs in many respects from those of other commercial publishers. First, they hire no salespersons and use no advertising agencies. Further, the materials may be adopted only with a commitment of staff development by a district. This staff development is supervised partially by CDA trainers.

### 2. Description of Materials

A brief description of CDA Math materials is given below. Books names "a" are for grade 1, "b" for grade 2, ... and "f" for grade 6. All student booklets are consumable. Student books a, b, and d are bilingual with instructions in both English and Spanish.

Student Booklets:

Individualized Computation, levels $a_1$, $a_2$, $b_1$, $b_2$, $c_1$, $c_2$, $d_1$, $d_2$, $e_1$, $e_2$, $f_1$, $f_2$ teach facts and operations, whole numbers and fractions.

My Progress Book abc; My Progress Book def. Tests included are identical to those found in the Individualized Computation books.

Patterns and Problems, levels a, b, c, d, e, f. There are two focuses: practice of facts and operations in problem settings and introduction of such topics as geometry, measurement, money and time.

Teacher Resource Books:

Mathematics for Everyone (This book is volume I of Drill and Practice at the Problem Solving Level.) It is a general overview to the philosophy, goals and materials of the program.

Drill and Practice at the Problem Solving Level. These are loose leaf pages for reproduction for use with students. Notes for teachers are on the reverse side. There are three levels of activities involving manipulative objects, sketches, and arithmetic notation.

Banking on Problem Solving. This is a two-volume format for grades 4-6. The first book is an introduction to the problem solving approaches and to the philosophy and theory of child development that underlies it. The second volume is a bank of activity pages with extensive comments to teachers.

Experiencing Fractions. This book contains suggestions for fraction activities using manipulative materials such as egg cartons and measuring sticks.

Placement Activities and Tests. This set of materials reviews formats used for computation and illustrates the types of activities for practice and problem solving. Its use permits the starting of pupils in CDA Math at any grade level 1-6.

Two supplementary resources are available to develop language and thinking skills through mathematics. The first, called Think, Talk and Connect, is designed for use at the kindergarten and first grade levels.

It consists of a box of duplicating masters and a teacher's guide. The second resource is the Think, Talk and Read Math Center. It provides for oral language experiences and language development by using activity cards that children "read". The teacher's guide includes a card-by-card commentary describing the focus, pre-activities, think and talk suggestions, story starters and related cards.

A special feature of CDA Math is the concerted effort to involve parents. Principals are given direct responsibility for building parent interest. A kit called Making Friends with Numbers is available for use by parents, teacher aides, student tutors, or others working on a one-to-one relationship with children. In addition, Individualized Computation books for levels a, b, and c contain letters to parents for each unit.

3. Inservice Education Required

An inservice program is required for introduction of CDA Math for teachers. It is highly recommended that resource teachers and principals also receive training. Personnel trained by CDA will conduct initial workshops prior to the opening of school and provide follow-up support as required throughout the year. A cadre of trainers is available in the State. Schools that are interested should contact their district educational specialist for mathematics.

Some of the features of this inservice program are unique. A minimum of 20 hours initial training is required for teachers. Principals have a vital role as a curriculum leader. Resource teachers, in addition to carrying out the inservice program during the year, visit classes, may teach demonstration lessons, and provide help with problems that may arise during the first year of implementation.

The training is needed to develop an understanding of some different ways of "looking at" a mathematical problem, including strategies that

promote experimentation by pupils, and to help pupils plan solutions, make up problems and search for interesting results. Further, most teachers will need help in learning how to teach an "oral development of language." They must recognize that the program is designed to help with the problem of individualization and that they must develop ways to organize the class to make maximum use of this feature.

4. Mathematical Content

The mathematical content of the Mathematics Program Guide (MPG) is covered well by CDA. The numbers and operations and measurement strands as well as an emphasis on problem solving are evident even by casual observation. The geometry content is present but is somewhat more difficult to locate because it is so often imbedded in problem situations. Language development is an essential ingredient of the program. Pages early in the program have little or no reading so the first goal is oral language development. New vocabulary is introduced as it becomes more meaningful in the context of an activity. The vocabulary of geometry such as parallel, perpendicular, triangles, and so on is taught regularly as part of language development in many lessons.

5. Experiences for Students

There are many experiences for students that provide practice in computation with whole numbers, decimals, and fractions and in measuring with various degrees of precision. Word problems and graphing activities for developing concepts are also found. The curriculum is based on a movement of experiences from manipulative to representational pictures to abstract symbols.

Major differences between CDA and other commercial series do exist. Some of them are described below.

a. There are many experiences with physical materials such as beans and beansticks.

b. Many problems are "open-ended." Every student is able to complete a significant part of each assignment while some students are challenged by an extension of the basic task.

c. After working on a set of examples, students are asked to make up their own problems. Such activities are not isolated examples but an integral part of the program.

d. Practice is often disguised and in an interesting form. Students practice as they try to discover an idea or meet the conditions of a problem.

e. Pictures are used to cement understanding and concepts. Too often, pictures for a computation are used to illustrate a computation in early instruction and then discarded.

f. Problem-solving behaviors are practiced continuously. Such behaviors as recording results of experiments, organizing data, completing tables, searching for patterns and investigating ideas are found throughout CDA. One of the difficulties confronting teachers new to CDA is in getting the maximum out of the problem-solving strand. They must avoid telling students yet give them the needed guidance for attacking a problem.

6. Teaching Strategies

One of the features of CDA Math is a psychology of learning or child development in which students progress from the manipulative level to the representational to the abstract. Throughout each level, students build a supply of remembered experiences, engage in problem solving and make

investigations on their own.

The materials reflect this approach to how children learn mathematics. The approach, however, forces every teacher to make changes in teaching strategies. The usual strategies of class discussion and independent student work are basic in this program. But most teachers must make adaptions in their teaching procedures if students are to obtain maximum benefits from a study of this program. The most important of these are discussed below.

a. The emphasis in mathematics education is shifted from developing skill to involvement in problem solving. Drill and practice in computation is not an end in itself but becomes an indispensable tool or a means to the end of solving problems. Students are not expected to have a complete set of computational skills before they develop higher level skills in problem solving and making independent investigations.

b. Talking about the work is an important part of any lesson. Students discuss their plans for solving a problem, how their plan may have changed as the solution progressed, best ways of recording solutions, etc.

c. The teacher in CDA Math is a resource person who offers guidance and encouragement but keeps the student working even when the "going gets rough". The teacher must help the student be the "driver" and avoid taking over.

d. In this program, students are encouraged to choose their own methods for solving the problem and to take the problem to their own levels of sophistication. It is the author's contention

that, if the students have the necessary background and problems are made interesting, they will work toward solutions in their own styles and at their own rates.

e. Student evaluation does not consist merely of paper and pencil tests. In this program, teacher observation of students' behavior as they solve problems gives more and better information than mere written answers.

The above description of strategies indicates the need for a dedicated teacher and one willing to change. The author of the program says (when discussing individualization), "Rather than increasing the burdens on teachers, this new approach ... offers greater opportunities for them to realize their professional potential and to meet the diverse needs of each individual learner. The purpose ... is to specifically help teachers ... initiate problem-solving activities that include a mixture of individualization and interaction (among teacher and students)."

7. Summary

Some of the goals of CDA Math as given on page 64, Mathematics for Everyone are the following:

a. Encouraging perseverence.

b. Putting the learner in the driver's seat.

c. Solving substantial problems.

d. Building a pride in the learner of what he already knows and what he finds out.

e. Helping the learner record that knowledge.

f. Providing opportunities and recognition for independent investigation.

g. Developing a friendliness with numbers and shapes.

Of all programs presently available, CDA Math is the one strongest in problem solving as outlined in the MPG. Its implementation in the classroom, however, requires a thorough inservice program and teachers who do not resist change.

Nearly every school can improve its mathematics program with CDA materials or ideas from it. Few schools can say they meet the problem-solving objectives of the MPG. Hence, incorporating any problem-solving activities into the curriculum would result in program improvement. So if a school has adopted books, it may plan a staff development program through which teachers study problem solving with CDA materials.

One research study of two ESEA Title III projects using CDA Math shows gains of about 1.5 months for each one month in school. However, adoption of CDA Math materials alone does not guarantee an effective curriculum. Teachers who are willing to invest time to learn the approach, who believe that problem solving is important and who are flexible enough to try new strategies are essential.

8. Bibliography

There is little information on CDA in the literature of mathematics education possibly because it is somewhere between the commercial publisher and the experimental program with respect to adoption procedures, advertising and other features. For further information schools should contact their district educational specialist for mathematics or write to:

Curriculum Development Associates, Inc.
1211 Connecticut Avenue, Suite 414
Washington, D. C. 20036

## 3. Developing Mathematical Processes (DMP)

Developing Mathematical Processes (DMP) is a complete K-6 mathematics program of 90 topics, each in a separate booklet. Booklets 1-40 are for grades K-2; 41-64 for grades 3-4; and 65-90 for grades 5-6. Each topic consists of the appropriate teacher's guide, student booklet, testing materials, and kits of physical materials and printed materials.

1. Development of Materials

The program was developed by the Wisconsin Research and Development Center and is published commercially by Rand McNally and Company. Funds for the program came from the federal government and the state of Wisconsin.

The program was prepared for use by all students and has been tried out in schools.

2. Description of Materials

For each grade level K-6 a complete kit is available. Each kit contains the following physical and printed materials for the teacher and 32 students:

a. Physical materials kit. All needed manipulative materials for 32 pupils.

b. Printed materials kit. Picture cards, gameboards, graph paper, and duplicating masters for pre-assessment tests.

c. Teacher materials kit. A Teacher's Guide with pupil pages and answers, a Teacher's Resource Manual with additional guidelines and pages for recording pupil performance.

d. Student materials kits. 32 consumable student booklets.

e. Student guide kits. 32 sets of three nonconsumable student materials in book form for grades 3-6.

3. Inservice Education Required

The developer and publisher have trained a number of coordinators who are available to assist districts in the implementation of this curriculum. Further, a copy of the DMP Resource Manual is provided in each kit of teacher materials. This Resource Manual provides a framework around which to organize a complete inservice or preservice program.

The mathematics in DMP is not radically different from any present curriculum. However, learning to use the many materials and learning strategies for helping children become inquiring, independent problem solvers should require instruction with persons who have considerable knowledge about this curriculum.

4. Mathematical Content

The mathematical content seems to cover that which is described in the Mathematics Program Guide (MPG) very well. Some of the important language of the guide, words such as "explore" and "investigate," are found consistently.

The content includes the arithmetic of rational numbers, geometry, and the elementary ideas of statistics and probability. Measurement is strongly emphasized.

5. Experiences for Students

In general DMP is an example of activity-based learning. Concept development emphasizes a 3-step sequence beginning with physical experience, moving to pictorial experience and culminating with the use of suitable symbolism. Technical vocabulary is introduced as needed but delayed until the pupil has learned the concept. The materials provide a variety of activities and require the use of different physical materials. At the lower levels of the program little reading is required. There are many problem-solving activities such as investigating, finding relationships, and examining and organizing data.

6. Teaching Strategies

Regular objectives, those that are to be checked for mastery, and preparatory objectives, those that will be checked for mastery later, are listed in the teacher's guide for each topic. Review objectives, those that maintain mastery, are included for many topics.

Another part of each Teacher's Guide is a Topic Inventory. This Inventory assesses the possible mastery of regular objectives. The teacher may use it as a basis for individualizing the emphasis with different pupils. The same Topic Inventory is recommended as a post-assessment to check on pupils' accomplishment of objectives.

The Teacher's Guide also contains a list of the materials needed, suggestions for student preparation, teaching suggestions, and suggestions on classroom management.

While no supplementary materials seem to be available presently, there are open-ended activities, optional problems and regular reviews through which the program can be individualized.

7. Summary

The developer of the program states that IMP uses a multisensory approach and places emphasis on concepts in familiar situations. The use of technical vocabulary is delayed until it is needed and many lessons are introduced by stories containing cartoon characters. One objective of IMP is to develop inquiring, individual problem solvers. There are many opportunities for the students to practice behaviors leading to achievement of that objective.

Potential users should, however, be aware of the following comment by a reviewer: materials may place additional workload on the teacher.

8. Bibliography

a. Lockard, J. D. (ed.). Twenty Years of Science and Mathematics Curriculum Development, The Tenth Report of the International Clearinghouse. College Park, MD: University of Maryland, 1977. On pages 283-286 is a short summary of the program.

b. Wisconsin Research and Development Center, 1025 W. Johnson Street, Madison, WI 53706. The developer of IMP has published a number of research studies and reports concerning the preparation and evaluation of materials. These are available by writing the Center. Some of these reports are listed in reference a. above.

c. Rand McNally and Company, Box 7600, Chicago, IL 60680. The publisher or local representative has brochures describing IMP materials.

d. National Council of Teachers of Mathematics. Mathematics Learning in Early Childhood, Thirty-seventh Yearbook. Reston, VA: the Council, 1975. On pages 277-279 is a short but excellent commentary on IMP materials.

## 4. Distar Arithmetic

Three packages, labeled I, II, and III, constitute the Distar Arithmetic program. Each package of 160 lessons is one year's work covering the skills of arithmetic for whole numbers and beginning work on operations with common fractions. Brochures describing Distar state that the program has been started at age 3, age 4, kindergarten, first grade, second grade, and even higher grades. In each case, the program was deemed successful.

### 1. Development of Materials

Distar programs were developed empirically through years of research for reading and language as well as mathematics. The commercial publisher, Science Research Associates (SRA), feels it is an effective tool to use in teaching basic skills and concepts in these areas. The authors, in preparing this instructional system, believe that each student can be taught and educators need to find ways of teaching so that each student masters each basic element before going on to more complicated concepts.

The publisher has prepared a Summary of Case Studies concerned with the effectiveness of the Distar Instructional System. The studies report data on the achievement of students with varying backgrounds and varying abilities. This summary reports "that the Distar Instructional System has successfully done the job it was intended to do."

### 2. Description of Materials

The program, now in its second edition, has the following components.

a. Student Materials

For each of Distar I, II and III, there are three books of about 100 pages each for each child. They are called Take-Home Books for I and II and Workbooks for III. Each book contains written activities that the children do as part of the structured

presentation of the lesson and as independent seat work. The books are consumable.

b. Teacher Materials

A teacher's kit includes the following:

(1) Teacher Presentation Books, three for Distar I, four for II, and two for III. These books tell the teacher exactly what to say and do, what responses to expect from children and what corrections to make for common errors.

(2) A Teacher's Guide for each of Distar I, II, and III has preservice and inservice sections. The first introduces the program, describes how to implement it and provides general teaching techniques. The inservice section describes new mathematical content as it is introduced and provides general information on presentations.

(3) Teacher's Take-Home Books (or Workbooks) are student books with answers.

(4) Group Progress Indicators provide a means of sharing progress of each group in the class.

The kit also contains form boards, acetate page protectors and decks of cards which are uded in teaching. This material is non-consumable.

3. Inservice Education Required

The preservice section of the teacher's guide may or may not be used with an experienced Distar trainer. An annual 3-day preservice event is held each summer in Hawaii. However, the teacher's guide is written so that the prospective teacher who is unable to attend these sessions may learn and practice with another person, preferably someone who will also be teaching Distar. Costs of special Distar training are negotiable with the publisher.

The inservice section is to be used throughout the year. As new skills are to be introduced they are described and teachers may practice an exercise before presenting it to the children.

4. Mathematical Content

This program is limited to arithmetic only. The content as identified in scope and sequence charts is very similar to that of most presently used programs (in arithmetic) and of the Mathematics Program Guide (MPG) except for some differences in language and grade placement of topics. Some sample topics from each book are listed below.

a. Distar I

First lessons are on rote counting and continue with rational counting, symbol identification, symbol writing, equality, addition (some sums of 18 studied), counting backwards, subtraction and story problems.

b. Distar II

The first lessons review Distar I for those entering the program. New topics include completion of addition and subtraction facts, addition and subtraction algorithm, multiplication and division facts (division is indicated as a multiplication, $3 \times \square = 6$), story problems, a little work with fractions (adding, reducing, and multiplying) and a few lessons on negative numbers.

c. Distar III

Practice with facts is continued. Algorithms for the four operations are studied along with considerable work with fractions and factoring. There are more story problems and some formal problem-solving procedures are reinforced.

In Distar II, there are a few lessons on finding area by counting squares, measurement of length and weight, telling time and coin problems.

The Learner Objectives of the MPG which concern arithmetic in grades K-2 are covered in Distar. Some of the objectives for measurement and almost none for geometry are covered. The problem-solving emphasis in the MPG identified in the Learner Objectives and the behaviors listed on pages 16 and 17 are not present in Distar. In Book III, formal problem-solving procedures are practiced. These are quite different from those in the MPG.

5. Experiences for Students

The first sentence in each Teacher's Guide reports, "The concept underlying Distar Arithmetic is that virtually all children can learn if we teach them carefully. The program, therefore, provides the kind of careful instruction that is needed to teach basic skills."

There are essentially two kinds of daily activities. In the first, students answer questions in unison and on signals from the teacher. The other activity is independent work on the Take-Homes, two-sided worksheets containing written problems and activities. There is one worksheet for each lesson.

While the structured aspect of the program predominates all others, there are built-in features that provide for individualization. (See Teaching Strategies for a description of the structured aspect.) Papers are corrected daily, resulting in remedial work for some pupils. Details on how to group children, change membership in groups and proceed with faster-moving students are outlined in the Teacher's Guides.

Each Teacher's Guide describes three distinctive features of Distar relative to experiences for students: (a) The tasks are structured as

simply as possible and sequenced so that students can detect similiarity of tasks; (b) Practice is controlled and realistic for students to master objective; and (c) The sequence of skills is controlled for necessary mastery in preparation for more complicated tasks.

6. Teaching Strategies

Teaching strategies have been discussed in previous sections because it is impossible to completely isolate content, experiences for pupils and methods.

The lessons described in the Presentation book are extremely formal, detailing precisely what the teacher is to do and say. Further, there is no variation from day to day which might allow students to ask questions, describe a solution, discuss a problem, show a series of steps to other students, and so on.

The following paragraphs describe three important aspects of Distar. They concern the nature of this approach which can be overlooked if the materials are examined briefly.

a. Verbal proficiency in arithmetic is developed. In this program, arithmetic is considered a special form of language. It therefore uses language as a vehicle to habituate mathematical responses. So it is expected that children repeating certain language patterns will establish connections to important concepts.

b. The idea of mastery is built into the program. Each task is developed by a series of small steps leading to achievement of that task. The teacher is told to present each task until students' responses are firm. This pattern of teaching a small step toward a specific goal is repeated on a number of successive days.

c. A single basic strategy for each type of problem is learned. Furthermore, alternative approaches to solving problems are discouraged.

It is recommended that instruction for Distar be provided in groups, preferably three. Sample schedules for groups are provided.

It is expected that about one hour and ten minutes would be spent on structured arithmetic on every available school day. The Teacher's Guide indicates that the independent seat work can be scheduled at any regular convenient period.

7. Summary

A judgment on the worth of Distar is difficult. Its reliance on mastery of small bits may not agree with a person's theory of how pupils learn. Its formal procedures may be distasteful. Its claim that all children master specific objectives may be doubted. These statements can be debated with arguments to support both positions.

Many teachers will feel uncomfortable, not only with the rigid control of the learning situation but also with the plan for flexible grouping, the one hour plus per day scheduled for arithmetic, the lack of geometry and other features. At the same time, the careful analysis of tasks provided by Distar will give every teacher ideas and insights into how pupils learn verbal skills of arithmetic. Certainly the highly structured approach is somewhat unique. Yet, this approach yields some effective ways to learn. Reports from users are favorable.

Distar may be compared to selected objectives and goals established by the Mathematics Program Guide.

| Emphasis in Mathematics Program Guide | Emphasized in Distar |
|---|---|
| Problem-Solving | Not in the sense used in the Mathematics Program Guide (MPG) |
| Learner Objectives for K-2 | For arithmetic, beyond MPG, Slightly for measurement. Very little for geometry. |
| Develop curiosity | No |
| Develop communication skills | Strong emphasis on verbal and writing skills. |

8. Bibliography


a. National Council of Teachers of Mathematics. Mathematics Learning in Early Childhood, Thirty-seventh Yearbook. Reston, VA: the Council, 1975. This yearbook contains a short but excellent review of Distar.

b. Science Research Associates, Chicago, IL (publishers of Distar). Additional brochures cited in this summary are available.

## 5. Individualized Mathematics Program (IMP)

The Individualized Mathematics Program (IMP) is an individually prescribed instructional system based on stated performance objectives which contains student assessment/diagnostic devices with correlated teaching materials.

1. Development of Materials

IMP is published by Educational and Industrial Testing Service, San Diego, California.

This program is designed for use with all students and has been tried out in schools.

2. Description of Materials

IMP is a complete kindergarten-adult program divided into seven different levels. These levels are:

level K - kindergarten

level 1 - grade 1

level 2 - grade 2

level A - grades 3-4

level B - grades 4-6

level C - grades 7-9

level D - grades 9-adult

This review will consider only level C, grades 7-9. All materials are in a kit containing:

a. Student Progress Chart and Student Objectives Checklist folder. These are used to present a profile of each student's strengths and weaknesses.

b. Mathematics Progress Report. This report is used to indicate to parents the instruction that the student has received and the progress made.

In addition, Tests of Achievement in Basic Skills (TABS) is available for use with IMP. TABS consists of criterion referenced survey tests that measure specific learning objectives; they are designed to help the teacher determine specific skill weaknesses. Test results are used for student placement in the IMP system.

3. Inservice Education Required

No inservice education is required.

4. Mathematical Content

IMP provides the teacher with the organization, materials, and diagnostic/assessment devices needed to individualize instruction within the classroom. The instructional material for level C is based on 64 stated performance objectives and student progress is determined relative to the stated objectives. Therefore, IMP materials provide for continual assessment of student deficiencies and prescription of instructional materials.

When compared to the Mathematics Program Guide (MPG), IMP has considerable coverage of the objectives stated for Numbers and Operations. In many cases the IMP objectives dealing with Numbers and Operations exceed those stated in the Mathematics Program Guide.

Although IMP does provide for instruction in the areas of Geometry and Measurement, the MPG is much more explicit and stronger with respect to the experiences that should be provided to the students. It must be pointed out that the majority of IMP instructional material is computational with very little attention given to experiences such as: investigation, discovery, construction, estimation, etcetera. In addition, the problem-solving emphasis in the Mathematics Program Guide identified in the Learner Objectives and

behaviors listed in pages 16 and 17 does not seem to be present in the IMP.

5. Experiences for Students

The introduction in the Teacher's Manual for level C states, "...The IMP thus represents a way of providing for wide ranges of achievement levels represented in classrooms and gives each student the opportunity, and responsibility, for progressing at his own rate. This rate is determined by the individual capacity of each child."

Therefore IMP provides a flexible approach to implementation of individualized instruction in the classroom.

The majority of instruction is from Math-Paks. A Math-Pak is an eight page individual instruction booklet prefaced with a statement of the performance objective toward which instruction is directed. Each Math-Pak contains:

a. a diagnostic pre-test designed to identify a student's specific difficulty in achieving the stated objectives.

b. five practice-problem lessons each of which is related to one of the areas of difficulty isolated by the pre-test. From 10 to 20 practice problems are present in each lesson.

c. a problem set covering all operations required to meet the stated objectives.

d. a post-test parallel in content to the pre-test.

Math-Paks may be assigned in small groups or individually. Students are assigned those Math-Paks which are associated with incorrect items on the Pre-assessment TABS, Once the Math-Pak pre-test is corrected, the teacher assigns only those lessons for which instruction and practice are needed.

6. Teaching Strategies

No specific classroom teaching strategy is recommended by IMP. Rather, IMP offers a system of management by which a teacher can individualize instruction. IMP follows the principles of individualized instruction which requires for each individual student a continual:

Assessment of progress

Diagnosis of learning difficulty

Prescription of instruction

In general IMP was designed to provide an organization in which the teacher teaches individuals while providing basic mathematics materials for the teacher to use. IMP also provides for continuous evaluation of student achievement (relative to IMP behavioral objectives).

The approach to using IMP, as suggested by the authors, is reflected in the following sequence:

(1) The teacher selects from among the IMP objectives those appropriate to the classroom context.

(2) The teacher pre-assesses and records student progress.

(3) The teacher fills-in Student progress chart and assigns objective(s) to the student.

(4) The students are assigned the appropriate Math-Pak.

(5) The students complete Math-Pak pre-test.

(6) The teacher prescribes lesson or lessons in Math-Pak for students to complete.

(7) The teacher gives students help individually or in small groups as students work on lessons.

(8) The student completes lessons and self tests.

(9) The students complete Math-Pak post-test to the satisfaction of the teacher.

(10) The students continue to work in other Math-Paks until all assigned objectives are accomplished.

(11) The teacher post-assesses and records student progress.

7. Summary

IMP is a self-contained instructional system designed to pinpoint the student's weaknesses and provide effective instruction.

The IMP was developed to help teachers implement a program of individualized instruction by providing them with detailed performance objectives describing the learning product, and by providing methods and materials organized to achieve those objectives.

As stated previously, IMP has considerable coverage of the objectives stated for Numbers and Operations in the Mathematics Program Guide.

Instruction in the areas of Geometry and Measurement is also present but is essentially computational in nature.

## 6. Intermediate School Mathematics Program (ISMP)

The Intermediate School Mathematics Program (ISMP) is a comprehensive 7-8 mathematics program contained in five books. Books I and II are the basic seventh grade program, Books III and IV the eighth grade program, and Book V may be used in either the seventh or eighth grade.

1. Development of Materials

This program is a product of the Intermediate School Mathematics Project at the Curriculum Research and Development Group of the University of Hawaii. It is appropriate for use at the upper elementary and junior high school levels.

The program has been developed over a period of years beginning first as a series of booklets and culminating in a set of five books for students. Field testing occurred from 1975 through 1980.

2. Description of Materials

ISMP is being published by the Research Corporation of the University of Hawaii.

This section will contain a description of the ISMP materials. All inquires should be directed to: ISMP, c/o Dr. Nancy Whitman, University of Hawaii, 1776 University Avenue, Honolulu, HI 96822.

The ISMP is a two-year instructional program contained in five books. Books I and II are the first-year course of study and contain the following units:

Elements of Geometry (Angles, Protractor, Compass, Symmetry, Polygons)
Computation I (Primary Facts, Numeration, Mental Mathematics)
Computation II (Secondary Facts)
Distance and Direction (Different Coordinate Systems)

Measurement I (Metric Linear Units, Addition and Subtraction of Decimals)
Measurement II (Metric Area Units, Multiplication and Division of Decimals)
Fractions I (Idea of a Fraction, Addition and Subtraction of Fractions with Common Denominators)
Fractions II (Equivalent Fractions: Addition, Subtraction, Multiplication and Division of Fractions)

Books III and IV are the second-year course of study and contain:

Ratio (Enlargement, Comparing Geometric Figures, Scale Drawings, Gears, Comparing Objects, Rates)
Integers (Addition, Subtraction, Multiplication and Division)
Proportion, Percent and Similarity
Statistics with Probability
Shapes and Patterns (Polygons and Polyhedra)
Elements of Algebra

Book V may be used for both years:

Computation III (Logarithms and Exponents)
Fraction III (Prime Factorization)
Measurement III (Volume, Mass, Capacity)

Activities are an integral part of the program. Content and approach are blended in order to assure greater understanding. The program encourages problem-solving behaviors and stresses the use of language as a vehicle for mathematical comprehension.

The accompanying teacher resources are: (1) Teacher's Annotated Editions, keyed texts with commentary on various instructional strategies; (2) Supplementary Materials Folders, prepared quizzes, tests, and supplementary worksheets; and (3) Instructional Support Kit containing game and flash cards and other manipulatives essential to the program.

3. Inservice Education Required

ISMP requires teacher inservice education prior to adoption and during program implementation. Arrangements for inservice training can be made through Dr. Nancy Whitman.

The inservice education is needed to develop an understanding of the different ways of "looking at" a mathematical problem. Further, most teachers will need help in learning how to teach an "oral development of language."

4. Mathematical Content

The mathematical content of the Mathematics Program Guide (MPG) is covered very well by the ISMP. The emphasis on the numbers and operations, measurement, and geometry strands are evident even by casual observation. Language development is an essential ingredient of the program.

The ISMP takes the position that mathematics consists not only of "end products" but also the "mathematical processes" that must be used in order to achieve the "end products." For that reason the content is developed in a spiral approach. Students build "interlocking" experiences of increasing sophistication. The ISMP is designed to extend experiences a student has already encountered.

The general areas of study contained in the ISMP include:

Seventh grade: Geometry, Computations, Coordinate Systems, Measurement, and Fractions

Eighth grade: Integers, Ratios, Geometry, Proportion, Percent, Similarity, Statistics with Probability, and Elements of Algebra

5. Experiences for Students

One of the general goals of the ISMP is indicated by the following taken from the ISMP teacher's manual.

"...(to) satisfy the needs of students for content which is interesting, enjoyable, challenging, and provides for success."

One of the most noticeable characteristics of the ISMP is that activities are a prominent learning medium. These activities form an integral part of the total program. These activities may include projects, experiments, games, tapes, etc. In some instances these activities will function as "organizers"

of the mathematical content and in others they provide the necessary reinforcement of ideas already learned.

In addition, the ISMP stresses the use of language as a vehicle for mathematical comprehension. The relationships of the physical world, language, and mathematical symbols are incorporated in the program development. The ISMP rejects the view that poor readers can learn mathematics in a language vacuum.

Finally, not only does the ISMP encourage "mastery" and in-depth study of basic skills and concepts but it also encourages problem-solving behaviors such as looking for patterns, making and verifying conjectures, estimating, and looking back at the solution and process.

6. Teaching Strategies

For each unit of study there is a list of learner objectives to be taught. These learner objectives are keyed to the pages in the student's textbook. Coupled with the learner objectives are teacher commentaries which are designed to assist the teacher in using the ISMP materials. These commentaries give page-by-page guidelines on how the unit of study should be taught.

One of the most noticeable aspects of the instructional strategy utilized by the ISMP is the organization of the instructional units within each of the five textbooks. Each unit of study is not always worked on continuously but rather is interrupted by the study of another unit which is not too dependent on or related to the unit just studied. This strategy of switching has these added advantages:

a. It provides a change of pace for everyone, teacher included.

b. It provides students an opportunity to develop their self-confidence. Students who were weak on the first unit, will not necessarily be weak on the new unit.

c. It provides students a "rest" or "incubation" period. Students will have had an opportunity to "mull over," consciously or not, materials in the first unit.

d. It provides within a school year, a "wood-carving" effect. Students return to the study of a unit at various times and "chip-away" at it until mastery is achieved.

The instructional format of the program makes it possible for the teacher to easily switch instructional units. The switching of units generates a particular rhythm and pattern of instruction. Suggested switching-patterns are provided in the recommended sequencing of the materials.

7. Summary

Some of the features of the ISMP are the following:

a. Local color is used in concept formation, reinforcement, and application when appropriate. For example, the concepts of *mauka* and *makai* are used in developing the concept of bearings.

b. Activities are a prominent learning medium. The activities form an integral part of the total program.

c. The program demands "mastery" and in-depth study of certain basic skills and concepts as well as develops problem-solving behaviors.

d. The program stresses language as a vehicle for mathematical comprehension.

When compared to the 7th and 8th grade program outlined in the Mathematics Program Guide, ISMP covers well the mathematical content for Numbers and Operations, Measurement, and Geometry. Further, the emphasis on application, the use of physical material in concept development, and the importance of drill play major roles in this program.

## 7. Mathematics Resource Project

The Mathematics Resource Project has produced five large books, each over 500 pages, described as topical resources for teachers for grades 5-9.

The Mathematics Resource Project was conceived to draw from the vast amounts of material available and to produce usable resources for teachers. From these resources, teachers would select classroom materials as needed emphasizing interesting drill and practice, concept building, problem-solving, laboratory approach, and so on.

These materials, then, are intended to supplement an adopted program.

### 1. Development of Materials

Materials for the Mathematics Resource Project were prepared at the University of Oregon. They are based on research supported by the National Science Foundation. The present preliminary editions are published by Creative Publications.

The names of the five resource books are:

1) Number Sense and Arithmetic Skills
2) Ratio, Proportion and Scaling
3) Geometry and Visualization
4) Mathematics in Science and Society
5) Statistics and Information Organization

### 2. Description of Materials

Four of the resource books are divided into five major sections. These are described below.

a. Didactics. These are papers which give information on learning theories, teaching strategies, diagnosis and evaluation, and goals and objectives.

b. Teaching Emphasis. This section stresses important areas such as calculators, applications, problem solving, and so on which help in teaching most topics. For each area, there is a rationale describing its importance.

c. Classroom Materials. This section includes paper and pencil worksheets, transparency masters, laboratory cards and activities, games, bulletin board suggestions and teacher directed activities.

d. Teacher Commentaries. Each Classroom Materials section is subdivided into mathematical topics such as concept of decimals, addition of whole numbers, ratio as a real number, lines, symmetry, and so on. Teacher commentaries appear before each of these subsections. They give a rationale for teaching the topic, suggest how to involve students, suggest alternate ways to introduce or develop the topic, and so on.

e. Annotated Bibliography. This section lists sources which were used to develop the resource.

The fifth book, Mathematics in Science and Society, contains the usual Didactics section. This is followed by six sections entitled Mathematics and Astronomy, Mathematics and Biology, Mathematics and Environment, Mathematics and Music, Mathematics and Physics, and Mathematics and Sports. Each has four subsections:

a. Introduction

b. Classroom Materials. This includes worksheets, activity cards, projects, investigations, and other types of activities.

c. Glossary

d. Annotated Bibliography.

3. Inservice Education Required

There is no inservice education required. However, the materials could form the bases for workshops in geometry, applications, or in problem solving.

4. Mathematical Content

Except for Mathematics in Science and Society which is essentially applications of many topics in mathematics, the content of the resource books is essentially that of most present programs. Materials to supplement any topic presently taught are readily found. However, there is no basis for comparing the mathematical content to that of the Mathematics Program Guide. This project does not intend that any topic is covered completely or that a strict sequential development of that topic is followed.

5. Experiences for Students

A study of student materials reveals the following features:

a. Materials are included at many levels of difficulty.

b. The materials include a variety of experiences such as investigations, experiments, collecting data, and drawing conclusions.

c. There are experiences for pupils who need to repair deficiencies as well as others for fast learners.

d. There are many games, some just for fun, some to improve skills, and some requiring discovery of a strategy.

e. There are many experiences that are recommended in the Mathematics Program Guide. These include mental arithmetic, estimation, problem solving, applications, searching for patterns, encouraging communication and many more.

f. Many pages promote organization of data so students may discover generalizations.

g. Many pages require the use of models, paper folding, and many other manipulatives.

h. There are many examples of applications not ordinarily found in most materials.

6. Teaching Strategies

The teacher will find many suggestions on how best to use the student pages. Classroom management systems are suggested. Strategies and suggestions on how to help pupils are included in the Didactics, Teaching Emphasis and Commentary to Teachers sections.

In general, these resources are more than just pages that may be reproduced and given to pupils. Effective ways to use the pages and reasons why these are effective is an integral part of each resource.

7. Summary

The flexibility and variety of the Mathematics Resource Projects materials make the resource books very worthwhile. They are intended as a supplement to any mathematics curriculum for grades 5 through 9. However, the teacher has an important role in deciding which material is appropriate for his/her students. The teacher oriented sections on strategies, teaching emphases, and so on, help provide background and insights for making such decisions.

8. Bibliography

Because these resources are only in a preliminary edition and were completed recently, there are no reports or reviews available. Copies of the materials can be purchased from: Creative Publications, P.O. Box 10328, 3977 East Bayshore Road, Palo Alto, CA 94303.

## 8. The Nuffield Project

One of the best known and most respected British materials in the United States is the Nuffield Project. Although there are a few books that can be placed directly in the hands of students, the Nuffield Project consists mainly of materials for teachers.

Many features of the Nuffield material can be incorporated into our classrooms. Among them are the organization of a mathematics laboratory and imaginative activities for it, the emphasis on applications, and the stress on giving students an opportunity to think.

### 1. Development of Materials

The Nuffield Mathematics Project was funded by the Nuffield Foundation, a private agency. It started in the middle 1960's and most of its materials were completed before 1970. The planning and preparation of materials was done by mathematics educators from all grade levels. Samples of the products in workshops for teachers were extensive.

The materials are now over ten years old. They are, however, not out-of-date. They were written as an early reaction to the extremes of structure and formality of the sixties. Their informal approach with a stress on active learning and a desire to "spark children's mathematical imaginations" by shifting the emphasis from our world to their world" can contribute to an improved curriculum in most schools.

Additional information and prices can be obtained from

School Department
John Wiley & Sons, Inc.
605 Third Avenue
New York, New York 10016

### 2. Description of Materials

The Nuffield materials described below can be used by a teacher or a school in the following ways.

a. As a basis for learning about how to teach selected topics in mathematics and finding examples of effective activities for students.

b. Experiences with which to supplement any existing program.

c. Building a program for which the objectives include those related to increasing the use of physical materials, problem solving and applications.

If a study of these materials is undertaken it should be through a series of carefully planned workshops. (This is expanded later in this report.)

It is not a program in the sense that we understand the term because there are few books for students and no structured scope and sequence. However, for the purposes described above it is an excellent resource.

The most important materials of the Nuffield project are the following.

a. I Do and I Understand. An introductory guide which describes the philosophy of the "program" and the needed organization within the school to implement it.

b. General Guides.

1) The Story So Far. An index to early guides.

2) Into Secondary school. For teachers of older students.

3) Math's for Everything. For teachers of infants.

4) The First Three Years. For teachers of the first three primary years.

5) The Later Primary Years . For teachers of the later primary years.

6) A Guide to the Guides.

c. Teachers Guides (called Core Guides).

1) Computation and Structure. Covers the content under Numbers

and Operations in the Mathematics Program Guide (MPG).

2) Shape and Size. Topics from Measurement and Geometry.

3) Graphs leading to Algebra. This topic is for children ages 10-13 with most examples concerned with graphing ordered pairs of numbers.

d. Weaving Guides. Booklets for teachers which give detailed instructions and activities for a single concept. Titles include Desk Calculators, Probability and Statistics, Computers and Young Children, Logic, and How to Build a Pond.

e. Check-up Guides. These check-ups of student progress replace traditional tests.

f. Student materials for grades 6-9. These are modules devoted to single mathematics topics. A course can be built-up by the teacher using as many, or as few of the modules as desired. The order of use is chosen by the teacher. Modules can be "slotted in" to any course and used to supplement or partially replace standard texts.

More information concerning the content of these materials is needed to adequately describe the Nuffield Project. The following will help portray the flavor and spirit of the project.

a. The materials contain illustrations of ways to teach selected concepts and skills.

b. The materials have an activity orientation. Students are somewhat self-directive in following written directions and considerable reading skills are required. The motto of the Nuffield Project is:

I hear and I forget.

I see and I remember.

I do and I understand.

c. The materials emphasize discovery by students.

d. The materials develop awareness of mathematics in the environment.

e. Mathematics is integrated with other subjects. Objectives related to language instruction are developed through reading directions and the writing of reports of experiments.

f. The materials attempt to humanize mathematics.

g. Students are asked to think for themselves.

h. The materials stress how to learn rather than what to teach.

3. Inservice Education Required

The Nuffield materials can be used as a basis for workshops for teachers who wish to initiate or expand an activity or laboratory program in mathematics. Details of a recommended workshop program are given in the book, Freedom to Learn (see Bibliography). Two important objectives of the workshops are for teachers to become conversant with the aims and ideals of active learning and to be convinced that students can learn mathematics through their own discoveries. At such workshops teachers would adapt the activities outlined in the Nuffield materials and develop others to supplement presently used books.

4. Mathematical Content

The Nuffield materials do not have a structured scope and sequence, student books for each grade, a complete list of objectives nor specific expectations for mastery at a certain grade level. For this reason a direct comparison to Learner Objectives of the Mathematics Program Guide is impossible. However, the many examples of activities and lessons in the Teacher Resource Guide leads to the conclusion that the major mathematical topics of the MPG are covered. There is particular strength in the measurement strand.

5. Experiences for Students

The Nuffield Project believes that the mathematics laboratory is an effective way of learning mathematics. Class discussion, oral and written

reports, paper and pencil practice, and planning solutions by students are also important aspects of the learning experience. Learning to communicate with mathematics is a major objective of both this project and the Mathematics Program Guide.

The use of open-ended questions, real-life experiences, and graphing are characteristic of the Nuffield materials. Experiences that provide the opportunity to practice problem-solving behaviors (pages 16-17 of the Mathematics Program Guide) permeate all mathematics topics. Those most often found are collecting data, organizing data in tables, sorting according to various criteria, searching for patterns, and making reports of experiments.

The Resource Guides provide the teacher with only a limited number of pages of drill. This need for drill is recognized but it is assumed that teachers can develop such lessons as needed.

6. Teaching Strategies

Many teaching strategies have been identified or implied in previous sections. Many of them have been related to teaching in a laboratory situation where students participate actively in activities, organizing data, drawing and checking conclusions, and preparing reports of their conclusions.

Many of the teaching strategies suggested promote a pupil-centered curriculum, the relation of a mathematical model to a real world situation, the importance of intuitive thinking, and the development of communication skills.

7. Summary

The Nuffield Project has, for the most part, produced materials for teachers. The student materials are on specific and, in general, independent topics. In England the entire set of materials would be used in

workshops for teachers and then by the teachers themselves as a reference. They could be very effective in an inservice course here and even more effective if the expressed purpose of the course was to develop laboratory experiences.

Individual teachers may or may not find the Nuffield materials useful. They may not be able to easily locate the specific topic they wish to supplement. Activities are sometimes organized (in the Teacher Guides) into a series of a few consecutive lessons. More often a variety of experiences with a set of materials is described.

A negative reaction to the Nuffield materials from the preceding paragraph is more than balanced by positive factors. This is evidenced by the fact that the answer to each of the following questions is, "Definitely yes."

a. Can certain activities of the Nuffield Project be incorporated profitably into our program?

b. Can teachers learn to teach a more activity-oriented program by studying these materials?

c. Can experiences be found to replace presently used "tell and do" activities?

d. Can activities be found that help students communicate mathematically and learn to attack problems?

Thus, most teachers will be able to find ideas to improve their lessons in mathematics and the faculty of a school or district can profit from a careful study of the Nuffield materials. However, that study should be planned in the form of a series of workshops with a leader familiar with the Nuffield materials.

8. Bibliography


a. Biggs, E. E. et al. Freedom to Learn. Don Mills, Canada: Addison-Wesley (Canada) Ltd., 1969. This is an excellent book describing all aspects of an active approach to learning mathematics.

b. Central Advisory Council for Education. Children and Their Primary Schools. London: Her Majesty's Stationary Office, 1967. Two volumes of over 1000 pages give a complete report of all aspects of British education up to age 13. The section on mathematics summarizes the status of teaching that subject.

c. Howson, G. (ed.). Children at School: Primary Education in Britain Today. New York: Teachers College Press, Columbia University, 1969. This deals with curriculum; contains an especially good chapter on changes in teaching mathematics.

d. National Council of Teachers of Mathematics. Mathematics Learning in Early Childhood, Thirty-seventh Yearbook. Reston, VA: the Council, 1975. A short but clear review of the mathematics program for British children ages 5-13.

e. Rogers, V. R. Teaching in the British Primary Schools. London: Macmillan, 1972. A fine description of practices in the primary schools with one chapter on teaching mathematics.

## 9. The School Mathematics Project (SMP)

The School Mathematics Project's (SMP) Basic Mathematics Series is intended for students from grades 7 to 10. It includes student texts with accompanying teacher's editions. Supplementary materials are also available, but not required.

### 1. Development of the Materials

SMP was founded in England to develop new mathematics courses which would adequately reflect the modern nature and use of mathematics. By 1967, it was clear that the content of SMP, with some modifications, was suitable for a much wider range of students than was originally planned. The result was the Basic Mathematics Series, Books A - H.

The series as well as other SMP materials is available in the United States through:

Cuisenaire Company of America, Inc.
12 Church Street
New Rochelle, NY 10805

### 2. Description of Materials

a. Student texts: Books A - H are softbound books. Topics include numbers and operations, geometry, probability and statistics, measurement, matrices and trigonometry. Topics are interwoven throughout the series with a spiral effect. Accompanying teacher's editions contain student pages and both answers and suggestions on how to teach the topics.

b. Supplementary materials: In addition to the basic set, the following are available for enrichment and/or teacher resources.

1. SMP Activity Cards: These cards contain material found in books A and B. Language is simplified and content is presented in smaller units. Three packs are available. The Main Pack Cards I is activity-oriented. It enables students to begin at different points and different levels, and to progress at their own speed.

   Preliminary Cards offer a review of basic operations of arithmetic. Supplementary Cards extend topics introduced in the Main Pack.

2. Elementary Tables: This little pamphlet contains three-place logarithm and trigonometric function tables and a list of helpful formulas. Tables of squares, square roots, and reciprocals are also included.

3. SMP Books X, Y, Z: This series extends topics introduced by A - H, thus allowing SMP to offer a 7-12 curriculum. Accompanying teacher's guides provide answers as well as background information to topics and suggestions on the treatment of these topics.

4. Other publications such as the Special Mathematics Series, Books 1 - 5 are also available. This series is intended for upper quartile students from grades 7 through 11, and is a resource set for the teacher. For a full listing of SMP publications and prices write to the Cuisenaire Company.

3. Inservice Education Required

There is no inservice education required. However, workshops and teacher training on SMP are available. Contact the Cuisenaire Company for more information. Since many of the topics covered in SMP are not found in more traditional programs, a teacher using these materials would find some sort of inservice information worthwhile.

4. Mathematical Content

The content of SMP addresses most of the Mathematics Program Guide (MPG) objectives. The goals of SMP are to make the study of mathematics exciting and enjoyable and to expose students to the nature of mathematics and its use in the world today. Aside from topics normally found in 7-8 grade texts, topics like vectors and matrices, transformations, trigonometry, three-dimensional geometry, and statistics and probability are integrated throughout the eight books in the series. Some of the objectives of Core Algebra, Selected Algebra Topics and Core Geometry are met by the Basic Series but SMP Books X, Y, Z must be used to complete the picture. (SMP Books X, Y and Z extend the topics introduced by A - H and are intended for Option Y type students; thus allowing SMP to offer a 7-12 curriculum.)

The emphasis of the program is towards mathematical ideas rather than manipulative techniques. Problem-solving behaviors like looking for patterns and making conjectures are fostered by the presentation of ideas.

5. Experiences for Students

Experiences for students are more varied in SMP than in other programs. Major differences between SMP and other programs include the following:

a. The spiral approach is used. Topics of arithmetic, algebra,

geometry, and so on, are interwoven throughout the program. In fact, it is often difficult to tell at any one time whether one is doing algebra, geometry, or some other area of mathematics.

b. Activities play a prominent part in concept development. "Teacher talk" is minimized, with students taking an active role in the learning process.

c. Problems do not always have a right answer. Instead, these problems lead to an observation, a judgement, or a 'best' possible answer (after some discussion).

d. Content is broken down into small "bites". This provides a change-of-pace type of scheduling. At the same time, student interest is maintained during each short chapter.

e. Mathematical content is drawn out of real situations which are easily recognizable, understandable, and genuinely interesting to students.

f. Development of problem solving skills is encouraged.

g. Mathematical ideas are first described in language which is not itself mathematical. Language should be a vehicle, not a barrier to mathematics.

6. Teaching Strategies

Because of the nature of SMP, the role of the teacher is not one of "information provider". Instead, the teacher is a guide and partner in the student's learning process. A classroom atmosphere which encourages peer teaching can yield a big pay-off. Every effort should be made to promote problem-solving behaviors, such as looking for patterns, making and verifying conjectures, looking at alternate methods for doing a problem, and so on.

Since there are many activities in SMP, classroom management problems may arise. Some suggestions and warnings are offered in the teacher's guide.

7. Summary

The goals of SMP are to make the study of mathematics exciting and enjoyable, and to reflect the nature of mathematics and its uses in the modern world. Its success towards these ends is evidenced by its widespread use in Great Britain (over 60% of all secondary students). It offers an integrated approach to mathematics, thus giving students an opportunity to meet the mathematics of the present.

8. Bibliography

a. Thwaites, B. The School Mathematics Project: The First Ten Years. Cambridge, Great Britain: Cambridge University Press, 1972.

ELEMENTARY AND INTERMEDIATE
SCHEDULE G
SUGGESTIONS FOR IMPLEMENTATION

The examples below provide suggestions for beginning the implementation of the school's improvement plan. These suggestions do not cover all the possibilities nor are all suggestions appropriate for every school.

1. Contact District Mathematics Coordinator regarding
   - deadlines for in-service requests
   - in-service needs identified in the school's improvement plan
2. Contact District Mathematics Coordinator regarding
   - availability of resource personnel (e. g., U. H. College of Education, community college, parent and community groups)
   - the nature and availability of District and State resources
   - the services of the District Mathematics Resource Teacher
   - examination copies of current textbooks series, alternative mathematics programs, and supplementary materials
   - assistance in using or reviewing instructional materials in the District's Resource Center.
3. Contact textbook representatives and publishers through District Mathematics Coordinator
   - availability of teacher in-service training
   - examination copies of instructional materials (e. g., for remedial, for enrichment, for incorporating problem-solving)
4. Begin setting up a teacher resource room or area
   - collect examination copies of student and teacher books
   - inventory manipulatives and games that are available at the school
   - subscribe to the Arithmetic Teacher and/or Mathematics Teacher
5. Schedule committee and faculty meetings related to mathematics for the school year
   - schedule meetings at periodic intervals
   - have a specific agenda for each meeting

6. Plan for a quarterly newsletter highlighting accomplishments, current tasks, and future goals relative to the school's improvement plan

7. Determine criteria for evaluating intervention strategies identified for improving the mathematics program

8. Establish in-house in-service training for particular content areas and/or teaching strategies

9. Plan for teachers to observe other teachers within the school

10. Plan on visitations to other schools

11. Contact schools within complex (elementary feeder schools, intermediate feeder schools, high schools) for articulation meetings

# High School Schedules A Through G

# 5. HIGH SCHOOL SCHEDULES

## Introduction

The flowchart for Assessment and Improvement Activities found on the next page represents an interpretation of the Foundation Program Assessment and Improvement System (FPAIS) for school initiated change in mathematics. The flowchart relates Schedules A through G to the assessment activities, planning for improvement, and implementation of the improvement plan.

The model assumes that the leadership team consists of an administrator and at least two mathematics teachers. Schools may find that having the mathematics department chairperson on the leadership team facilitates the coordination of the assessment and improvement activities.

The guidelines offer suggestions to the leadership team directing the school-level mathematics assessment and improvement activities. These guidelines are based on the experiences of schools that were involved in piloting this model.

Schedules A, B, and C help to organize the assessment activities. These schedules compile the available data relative to the mathematics department's budget, personnel, student test results, and so on. The schedules highlight recent developments such as the two-credit mathematics requirement for graduation, the Hawaii State Test of Essential Competencies (HSTEC), and the Mathematics Program Guide. The school and department profiles provided by these schedules serve as a basis for school-level planning for improvement. For example, by identifying the number of students taking less than two credits of mathematics on Schedule A, the mathematics department can begin planning for accommodating these students.

Schedules D and E are related to the school's planning for improvement. This process begins with the leadership team analyzing the information on the

HIGH SCHOOL
FLOWCHART FOR ASSESSMENT AND IMPROVEMENT ACTIVITIES

Leadership Team organizes available data related to school and department on Schedule A: School and Department Data, Schedule B: Testing and Student Evaluation and Schedule C: Post High School Information

↓

Team analyzes problem areas reflected in completed Schedules and other sources to complete first draft of summary (Schedule D: Summary)

↓

Team leads mathematics department in discussion, analysis, revision of Schedules A, B, C and first draft of Schedule D

↓

Team/mathematics department completes Schedule D based on this input

↓

→ Team/mathematics department identifies and prioritizes needs on Schedule E: Planning for Improvement
Team begins first draft of improvement plan and timelines on Schedule E
Team utilizes appropriate sections of Schedule F: Materials Assessment

↓

Team leads department meeting to analyze and revise improvement plan and timelines

↓

Team/mathematics department completes improvement plan and begins implementation using Schedule G

↓

Team coordinates and monitors department implementation of improvement plan

↓

Team/mathematics department reassesses needs and evaluates improvement plan (including adjustments, revisions, etc.)

↓

Team/mathematics department reviews and updates Schedules A, B, C, and D

assessment Schedules A, B, and C, including their current course offerings relative to their student population, budget, and staff inservice training needs. The leadership team also reviews the appropriate parts of the Manual, including the trends and issues section and Schedule F for improvement alternatives. following input from the department members, a three-year improvement plan is finalized on Schedule E.

The suggested timeline is to complete the improvement plan within one school year. Moreover, it is strongly recommended that the team begin implementation of the plan to a point where they will be able to continue the momentum for the next year. Suggestions for beginning implementation are provided in Schedule G.

Schedule A School and Department Data

The intent of Schedule A is to provide a profile of the school and the mathematics department: its student population, professional staff, course offerings, administrative policies and procedures, and teacher awareness of the Department of Education's goals and priorities for mathematics education.

This information serves as a basis for analyzing data on subsequent schedules and for formulating the school's improvement plan. For example, the data on the type and number of courses the department currently offers may be compared to the HSTEC results on Schedule B or the post high school plans of twelfth graders on Schedule C.

The leadership team completes Schedule A. The completed schedule may be used to involve the rest of the department. Some teams may decide to complete Schedules A, B, and C before involving the department.

The Principal has access to most of the data at the school. Other data may be found in records kept by the mathematics department chairperson, the school registrar, or counselors.

Schedule B Testing and Student Evaluation

Schedule B compiles the results of the various tests that are administered statewide to high school students. It provides a profile of student performance in those areas covered by the test. The results of tests included on Schedule B are available at the school. Test results are usually kept by the registrar, the counselors, or the adminstrator responsible for testing. Analysis of test results should provide the leadership team with information on strengths and weaknesses in the school's program.

The Department of Education has identified fifteen Essential Competencies which are included in the objectives and expectations section of the Manual. The leadership team may want to call the School Testing Coordinator for help in reading and interpreting the school's HSTEC results. These results may influence the department's assessment of the content and skills to be taught in the Option X courses.

Schedule C Post High School Information

Schedule C is designed to summarize the data that are available on high school graduates. The intent is to help the leadership team analyze the mathematics program in terms of how well the needs of the students were met.

Most of the data, including the grade distribution from the University of Hawaii at Manoa, are available at the school level, usually in the office of the registrar or the counselor. A similar grade distribution of incoming freshmen is available from the University of Hawaii at Hilo and each community college in the system. Leadership teams that determine a need for this information may contact the registrar of the respective campus.

The completed Schedules A, B, and C provide a means of sharing relevant information with the other mathematics teachers, as well as a means for beginning the analysis of the school's mathematics program. For example, a comparison of the data reflecting the intentions of twelfth graders on

Schedule C with the department's program of courses on Schedule A may reveal student groups whose needs are not being met.

Other considerations in the analysis include requirements in mathematics for admission to colleges and universities as well as for particular fields of study. Counselors and guidance teachers usually have the latest available information.

Schedule D Summary

Schedule D organizes and summarizes the analysis of the mathematics program. This process involves identifying the areas of improvement.

The analysis is conducted by the leadership team and/or the mathematics committee based on the needs reflected in Schedules A, B, and C. The goals and objectives of the Mathematics Program Guide, the course guides from the Department of Education, college mathematics requirements and the information in the Manual on trends and issues, problem solving and reading in mathematics may also suggest need areas.

The completed first draft of Schedule D, along with completed Schedules A, B, and C, serves as a means of promoting dialogue and articulation within the department. Some leadership teams may decide to complete the schedules up to this point before involving the other members of the department in the assessment and improvement activities. Schedule D is finalized with input from department members, and is used to draft the improvement plan on Schedule E.

Schedule E Planning for Improvement

Schedule E provides a format for developing the department's three-year improvement plan. The needs identified on Schedule D are analyzed with respect to resources required and constraints imposed. Modifying, adding, or deleting courses to reflect the Mathematics Program Guide are highlighted.

The leadership team and/or the mathematics department may choose to review the information in the Mathematics Program Guide, the Manual, and the

available course guides in planning for alternatives and intervention to address the department's needs. The mathematics department may also request assistance from District, State, University or other resource personnel during their planning.

The draft of the improvement plan including the time frame suggested on Schedule E is shared at a meeting for reaction and input from the mathematics faculty. The improvement plan is revised and finalized based on this input. The support and involvement of the department members are necessary components in the implementation of the school's improvement plan.

An example of the projected timeline is provided on page 5-26 in Schedule E. It illustrates some of the steps involved in developing and implementing a new course.

It is strongly recommended that the leadership team begin the first steps toward implementation of the improvement plan without a lapse in time. Generally the team has organized into a cohesive working group and the department has been involved in the assessment and improvement activities. This situation facilitates the implementation of the improvement. Also, a retracing of steps that may be required after a vacation, slows the momentum that had been established. A few suggestions for beginning the implementation are provided in Schedule G.

Schedule F Materials Assessment

Schedule F contains a form for each course in the Mathematics Program Guide for which specific learner objectives are given. Each form is designed to help in the selection of materials for modifying or developing the course. While the forms focus on the learner objectives outlined in the Mathematics Program Guide, other factors such as reading level of the materials and availability of supplementary materials are considered.

Schedule F may also be used to assess materials being used in courses the Department currently offers. It provides a means to familiarize mathematics teachers to the goals and objectives of the Mathematics Program Guide.

Schedule G Suggestions for Implementation

Schedule G provides a few suggestions that may be appropriate for the first few steps in implementing the school's improvement plan. These initial steps are intended to help "get the ball rolling".

The suggestions do not cover all possibilities nor are all suggestions appropriate for every school. Rather, several sources that schools may contact for help in implementing their plan and examples of intervention within the school are provided.

HIGH SCHOOL
SCHEDULE A
SCHOOL AND DEPARTMENT DATA

I. School Data

A. Circle grades serviced: 7 8 9 10 11 12

B. Student enrollment (School Year 19__ to 19__): _____

C. Name of Principal: _____

D. Name of Vice-Principal(s): _____

E. Number of regular classroom teachers: _____

F. Number of other professional personnel:

_____counselors ____ other off-ratio teachers

_____librarians

_____student activities coordinators ____others (specify)_____

G. Auxiliary programs related to mathematics (e.g., Title I programs)

_____

II. Mathematics Department Data

A. Total mathematics enrollment (School Year 19__ to 19__) _____

B. Number of mathematics classes (School Year 19__ to 19__) _____

C.

| Name of Teacher | Certified to Teach Secondary Mathematics? | Mathematics Courses Taught |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

D. By the completion of their high school education, approximately what percent of students have taken only one credit of mathematics?_____ 2 or more credits?_____

1. How many additional mathematics courses/sections will be necessary with the new two-year mathematics requirement for high school graduation? _____

2. How will the mathematics staff size be affected by the new two-year mathematics requirement for high school graduation?_____

_____

E. Is there a resource/workroom for the mathematics department? If yes, is it used for

_____ department meetings
_____ coffee breaks
_____ source of current mathematics materials and periodicals
_____ work preparation
_____ informal meetings/discussions
_____ library of supplementary materials
_____ housing duplicating materials and supplies
_____ other (describe)

F. Use the most recent Authorized Courses and Code Numbers (ACCN) to complete the table below. Indicate the number of students enrolled in each course and the number of sections offered this school year.

| CODE NUMBER | COURSE TITLE | NUMBER OF STUDENTS | NUMBER OF SECTIONS |
|---|---|---|---|
| | GRADES 7 AND 8: | | |
| | | | |
| | OPTION X: | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

| CODE NUMBER | COURSE TITLE | NUMBER OF STUDENTS | NUMBER OF SECTIONS |
|---|---|---|---|
| | OPTION Y: | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | OPTION Z: | | |
| | | | |
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| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | FOR STUDENTS IN SPECIAL EDUCATION | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | COURSES TO BE PHASED-OUT (per ACCN) | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

III. School Budget

A. Is the school's overall fiscal situation (budget amount and procedures) discussed with the staff? _____

B. Budget allocation for previous two years, current school year, and the next year.

| SCHOOL YEAR | TOTAL SCHOOL | MATHEMATICS DEPARTMENT | OTHER RESOURCES ALLOCATED TO DEPT. (e.g., paper, ) |
|---|---|---|---|
| 19__ to 19__ | _____ | _____ | _____ |
| 19__ to 19__ | _____ | _____ | _____ |
| 19__ to 19__ | _____ | _____ | _____ |
| 19__ to 19__ (projected) | _____ | _____ | _____ |

C. Additional funding for previous two years, current school year, and the next year (e.g., Special Needs, Title I, Title IVB, etc.)

| | | |
|---|---|---|
| 19__ to 19__ | _____ | _____ |
| 19__ to 19__ | _____ | _____ |
| 19__ to 19__ | _____ | _____ |
| 19__ to 19__ (projected) | _____ | _____ |

D. Breakdown of mathematics department budget for previous two years, current school year, and the next year

| SCHOOL YEAR | EQUIPMENT | TEXTBOOK | SUPPLIES | OTHER |
|---|---|---|---|---|
| 19__ to 19__ | _____ | _____ | _____ | _____ |
| 19__ to 19__ | _____ | _____ | _____ | _____ |
| 19__ to 19__ | _____ | _____ | _____ | _____ |
| 19__ to 19__ (projected) | _____ | _____ | _____ | _____ |

IV. Department Procedures

A. Budget

1. How is the annual mathematics department budget breakdown determined?

_____

2. What criteria is used in allocating money within the mathematics department? _____

_____

B. Texts and Instructional Materials

1. How are texts and instructional materials selected?

_____ by committee _____ by department chairperson

_____ by course teacher _____ by department faculty

_____ other

C. Courses

1. What is used to determine the program of courses?

_____ test results _____ student needs

_____ enrollment projections _____ other (specify)

D. Feeder Schools

1. What feeder schools do you service? _____

2. What student and curriculum information is provided to the mathematics department by feeder schools? _____

_____

3. How is this information used by the mathematics department? _____

_____

_____

E. Transfers

1. Approximately how many students transfer into the department during the year?_____

2. What information is provided on students transferring into the mathematics department?

_____ previous mathematics courses _____ testing results

_____ previous mathematics grades _____ other

3. How is the information obtained used by the mathematics department?

_____

_____

V. Department Meetings

A. How often are mathematics department meetings held?____ Average length____

B. Has the school mathematics program been discussed and studied in relation to:

1. the Foundation Program Objectives?_____

2. the Essential Competencies (including HSTEC results)?_____

3. the Performance Expectations?_____

4. the Mathematics Program Guide (including the incorporation of broad goals such as problem solving and communication skills into the curriculum)?_____

C. During the last two years, which of the following items, in addition to those given in B. above, have been discussed in department meetings?

_____1. Communicating mathematics department concerns to the administration

_____2. Communicating mathematics department concerns to the counseling staff

_____3. Communicating mathematics department concerns to the other departments

_____4. Evaluation of previous year's mathematics program

5. Sharing of expertise and experiences gained from participation in:

_____a. inservice workshops/seminars

_____b. experimental/pilot studies

_____c. activities of professional organizations (such as HCTM)

_____6. Changes in mathematics program

_____7. Implementation of new courses

_____8. Textbook selection

_____9. Teacher work assignments

____10. Department budget

____11. Accomplishments of the mathematics department

____12. Other (desribe) _____

VI. Special Student Needs

A. Who provides counseling to students regarding mathematics requirements for career choices, college requirements, and graduation?

_____1. counselors _____3. mathematics teachers

_____2. guidance teachers _____4. other (describe)_____

B. How is this procedure carried out?

_____ in a systematic way

_____ informally

_____ other (describe)

C. What methods does the department recommend for meeting the needs of students who are

1. poor or non-readers? _____

2. non-English speaking students?_____

3. recent immigrants?_____

4. mainstreamed handicapped students? _____

5. other slow learners? _____

D. How are better mathematics students being challenged?

1. mathematics league _____
2. mathematics club _____
3. advanced placement classes _____
4. early admissions _____
5. enrichment of regular classes _____
6. individual student adaptive programs _____
7. acceleration in grades _____
8. other _____

HIGH SCHOOL
SCHEDULE B
TESTING AND STUDENT EVALUATION

## I. Test Information

A. Is there a master list of school test results?_____

B. Who maintains this list?_____

C. Who has access to this list?

_____ district personnel
_____ administrators
_____ counselors
_____ teachers
_____ other (describe)

D. Who communicates school results to the faculty?

_____ principal
_____ vice-principal
_____ counselors
_____ district personnel
_____ other (describe)

E. Test results from the most recent testing

List below test results obtained from each of the following:

1. Grade 8 Stanford Achievement Test for high schools with grades 7-12

| | Stanines 1-3 | 4-6 | 7-9 |
|---|---|---|---|
| Mathematics Concepts | ____% | ____% | ____% |
| Mathematics Computation | ____% | ____% | ____% |
| Mathematics Applications | ____% | ____% | ____% |
| Total Mathematics | ____% | ____% | ____% |

2. Grade 9 DAT

| | Stanines 1-3 | 4-6 | 7-9 |
|---|---|---|---|
| Numerical Ability | ____% | ____% | ____% |
| Abstract Reasoning | ____% | ____% | ____% |
| Space Perception | ____% | ____% | ____% |
| Verbal and Numerical | ____% | ____% | ____% |

3. Grade 10 Stanford Achievement Test of Academic Skills (TASK)

| | Stanines 1-3 | 4-6 | 7-9 |
|---|---|---|---|
| Mathematics Test | ____% | ____% | ____% |

4. HSTEC (complete by grade levels)

| | Basic Skills % pass | Other Life Skills % pass | Total % pass |
|---|---|---|---|
| Grade 9 | ____ | ____ | ____ |
| Grade 10 | ____ | ____ | ____ |
| Grade 11 | ____ | ____ | ____ |
| Grade 12 | ____ | ____ | ____ |

5. Scholastic Aptitude Test

| Mathematics Scores | Number | % |
|---|---|---|
| 200 - 299 | ____ | ____ |
| 300 - 399 | ____ | ____ |
| 400 - 499 | ____ | ____ |
| 500 - 599 | ____ | ____ |
| 600 - 699 | ____ | ____ |
| 700 - 800 | ____ | ____ |

What percent of the school's total population take the Scholastic Aptitude Test? ____

6. CEEB Mathematics Achievement Test Results

| Scores | Number | % |
|---|---|---|
| 200 - 299 | ____ | ____ |
| 300 - 399 | ____ | ____ |
| 400 - 499 | ____ | ____ |
| 500 - 599 | ____ | ____ |
| 600 - 699 | ____ | ____ |
| 700 - 800 | ____ | ____ |

What percent of the school's total population take the CEEB Mathematics Achievement Test? ____

F. List other ways in which your school or department assesses interests, attitudes, and achievement of its students in mathematics.

G. Describe how these tests and assessments have been used in:

1. placing students _____

_____

2. determining future course offerings _____

_____

3. assessing mathematics department performance _____

_____

4. other _____

_____

H. Do tests of the past few years indicate an improvement or a decline in the students' mathematics knowledge? _____

_____

HIGH SCHOOL
SCHEDULE C
POST HIGH SCHOOL INFORMATION

I. Intentions of Twelfth Grade Students* (Class of 19__)

| | 4-year College | 2-year College | Trade-Technical School | Work | Military | Other | Undecided |
|---|---|---|---|---|---|---|---|
| Number | | | | | | | |
| Percent | | | | | | | |

II. How is the information provided by the annual statewide study of graduates utilized in improving the mathematics program?

_____

_____

III. How are the grade distribution data provided by the University of Hawaii at Manoa utilized in improving the mathematics program?_____

_____

IV. A. What other type of feedback has been provided the mathematics department by the following:

Community colleges_____

University of Hawaii_____

Other colleges and universities_____

Students themselves_____

Parents_____

Other_____

B. What effect, if any, has this feedback had on the mathematics department?

_____ change in offerings

_____ change in program

_____ change in teaching style

_____ no effect

_____ other (describe)

*This data is generally available in the most recent school accreditation report.

HIGH SCHOOL
SCHEDULE D
SUMMARY

Schedule D should reflect the current status of the mathematics department in terms of how well the curriculum, staff, instructional materials, and so on, are meeting student needs.

Use the information on Schedules A, B, and C, the policies of the school, district or state that are related to mathematics education, your own experiences and relevant information from other sources to complete this schedule.

I. Course Analysis

To complete this section refer to the course listing completed in part II.F. of Schedule A for courses that are being offered. Also refer to the curriculum guidelines and course descriptions in the Mathematics Program Guide (MPG).

A. Courses to be Continued

List below courses that:

--are currently being offered and
--reflect the course descriptions in the MPG and
--will continue to be offered in essentially the same format.

B. Courses to be Modified

In the table below:

1. List courses that will require a limited number of changes to reflect the course description in the MPG.

2. Identify change(s) for each course (e.g., change from year to semester course, topics to be included or omitted).

3. Indicate the order in which the department will begin these course modifications.

| COURSES TO BE MODIFIED | | |
|---|---|---|
| Course Name | Change(s) | Order |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

C. Courses to be Dropped

List those courses that are to be phased out according to the ACCN.

D. Courses to be Added

1. List new courses that may be appropriate for the department to offer. These selections may be based on considerations such as

   a. needs of students who are currently taking less than two years of mathematics.

   b. interests of students relative to new course opportunities.

2. Indicate the order in which the department should begin to implement these new courses.

| COURSES TO BE ADDED | |
|---|---|
| Course Name | Order |
| | |
| | |
| | |
| | |
| | |
| | |

II. Analysis of Other Areas

List additional aspects of the mathematics program or department that require intervention relative to:

1. administrative procedures (e.g., budget, school liaison)

2. curriculum/program concerns (e.g., instructional materials, student needs)

3. staff (e.g., inservice needs)

4. other

HIGH SCHOOL
SCHEDULE E
PLANNING FOR IMPROVEMENT

I. Analysis of Needs

To complete this section, refer to responses made in Schedule D.

A. For each course that needs modification:

1. Identify resources that will be required (e.g., materials, services, funding).

2. Identify constraints (e.g., time, funding, personnel).

3. Identify additional information/data required before changes are made.

4. Estimate a reasonable timeline for planning, implementing, and evaluating the course modifications decided upon.

B. For each new course that the department has decided to implement:

1. Identify the needs of students being addressed (see Schedule D, part I.D.1.).

2. Indicate what needs to be done before implementation (e.g., provide a division of responsibilities).

3. Identify resources that will be required (e.g., materials, personnel, and funding).

4. Identify constraints (e.g., time, funding, staffing).

5. Estimate a reasonable timeline for the development, implementation, and evaluation of the course.

C. For each of the additional needs identified in Part II of Schedule D:

1. Identify additional information/data required for clarification or decision-making.

2. Identify resources that will be required (e.g., materials, services, funding).

3. Identify constraints (e.g., time, personnel, funding).

4. Estimate a reasonable timeline for each component.

II. Determination of Timeline

A. Listing of Priorities

1. Refer to the needs identified in Part I above. Determine those that the department will address during each of the next three years.

2. List these in the appropriate space below.

3. Prioritize the needs within each school year.

School Year 19__-__

School Year 19__-__

School Year 19__-__

B. Establishing Time Frame

1. Record priorities for each year on timeline. (Make three copies of this page and use one copy for each year.) Each priority should include a breakdown of all work that must be completed for implementation.

2. Estimate the amount of time necessary to accomplish each section of these priorities.

3. Indicate on the timelines when each of those sections will be accomplished during the school year.

Note: An example is given on the next page to illustrate the projected timeline for one priority.

| | YEAR: _____ | | | |
|---|---|---|---|---|
| | Qtr 1 | Qtr 2 | Qtr 3 | Qtr 4 |
| Priority 1 | | | | |
| Priority 2 | | | | |
| Priority 3 | | | | |

## Sample Timeline

In this example the development, implementation, and evaluation of Option x courses is projected over three years. Since most schools will probably work on more than one priority in any given year, only the appropriate section for each year is shown below.

| | YEAR: 1980-81 | | | |
|---|---|---|---|---|
| | Qtr 1 | Qtr 2 | Qtr 3 | Qtr 4 |
| Priority 1 | | | | |
| Priority 2: Examination of materials for Option x courses | | | | |
| 1. Obtain and examine available resources | | | | |
| a. DOE curriculum resources/materials guide | | ○ | ● | |
| b. DOE sample Selected Applications courses for both Level A and Level B | | ○ | ● | |
| c. Textbooks | | ○ | ● | |
| 2. Meet with district resource personnel for input | | ○ | ——— | ● |
| 3. Share materials and ideas within department | | ○ | ——— | ● |
| 4. Form two committees (Selected Applications A, Selected Applications B) to develop courses in 1980-81 | | | | ○● |
| Priority 3 | | | | |

| | YEAR: 1981-82 | | | |
|---|---|---|---|---|
| | Qtr 1 | Qtr 2 | Qtr 3 | Qtr 4 |
| Priority 1: Develop Option x courses | | | | |
| 1. Develop course curriculum for Selected Applications Level A | ○ | ● | | |
| 2. Develop course curriculum for Selected Applications Level B | ○ | ● | | |
| 3. Discuss courses with entire department, district resource person | | ○ | ——— | ● |
| 4. Provide explanation of courses to students, parents, other teachers | | | ○ | ● |
| 5. Purchase texts and supplementary materials required | | | | ○● |
| 6. Determine how courses will be evaluated | | | | ○● |
| Priority 2 | | | | |
| Priority 3 | | | | |

| | YEAR: 1982-83 | | | |
|---|---|---|---|---|
| | Qtr 1 | Qtr 2 | Qtr 3 | Qtr 4 |
| Priority 1: Implement Option x courses | | | | |
| 1. Implement courses | ○ | ——— | | ● |
| 2. Provide on-going evaluation for possible modification | ○ | ——— | | ● |
| Priority 2 | | | | |
| Priority 3 | | | | |

HIGH SCHOOL
SCHEDULE F
MATERIALS ASSESSMENT

Each of the Materials Assessment forms is designed to help in the selection of materials for modifying or developing a course. The forms should help the reviewer address questions of importance relative to that course. To compare a number of different materials, individual assessment forms can be completed for each review. To get a more complete analysis of any one material, assessments can be completed by different people for comparison.

The learner objectives outlined in the Mathematics Program Guide should be the primary focus in analyzing the materials for each course. The forms will help to focus on these learner objectives. Since they are considered minimum objectives, other topics may be included to supplement or to enrich the course.

As materials for a course are examined, place a check (✓) in the appropriate 'Learner Objective Covered' box if sufficient coverage is provided. The comment sections should be used to record reactions to the content of the materials relative to each objective. Comments might include things such as: insufficient coverage because--; coverage too theoretical; good development; used complex symbolism; many good examples provided.

In addition to correspondence with the Mathematics Program Guide, there are other considerations that may influence the selection of materials. These include reading level of the material, sequencing of topics, availability of supplementary materials, and so on. To help the reviewer focus on these and other considerations, questions have been included at the end of the learner objective listings on these concerns. Additional questions that are of concern to your mathematics program can and should be added where appropriate.

LEVEL A

| Title _____<br>Author _____<br>Publisher _____<br>Copyright Date _____<br>Cost _____ | Learner Objective Covered | Comments |
|---|---|---|
| NUMBERS AND OPERATIONS:<br>• Operations with whole numbers, fractions, and decimals | | |
| • Simple equations in one unknown | | |
| • Relationship between common fractions and decimal fractions | | |
| PLANE FIGURES AND SOLIDS:<br>• Triangles, squares, rectangles | | |
| • Common solids | | |
| • Length, area, volume | | |
| MEASUREMENT:<br>• Metric and customary rulers | | |
| • Problems involving measurement | | |
| • Perimeter and area | | |
| STATISTICS AND PROBABILITY:<br>• Construction and interpretation of graphs | | |
| • Reading charts, maps, graphs | | |
| • Collection and classification of data | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:

   a. sequencing of topics?

   b. reading level?

   c. use of technical language?

2. Does the teachers' guide include:

   a. strategies for teaching?

   b. supplementary activities?

   c. suggested time schedule?

   d. testing program?

   e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:

   a. problem solving strategies?

   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

LEVEL B

Title _____

Author _____

Publisher _____

Copyright Date _____

Cost _____

| | Learner Objective Covered | Comments |
|---|---|---|
| NUMBERS AND OPERATIONS<br>• Ratio, proportion, and percent problems | | |
| • Operations with integers | | |
| • Simple equations | | |
| • Relationship among common fractions, decimals and percents | | |
| • Properties of whole numbers | | |
| • Square roots of non-negative numbers | | |
| PLANE FIGURES AND SOLIDS<br>• Properties of similarity and congruence in construction and applications | | |
| • Length, perimeter, area, volume | | |
| • Three basic trigonometric ratios | | |
| • Pythagorean relation | | |
| MEASUREMENT:<br>• Measurements using appropriate tools | | |
| • Conversions within the metric system | | |
| • Perimeter, area, volume | | |
| STATISTICS AND PROBABILITY<br>• Preparation and interpretation of simple statistically-oriented graphs | | |
| • Collection and organization of data to find the mean, median, mode, and range of the data | | |
| • Decision-making on basis of data | | |
| • Counting principles and strategies | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

| Title<br>Author<br>Publisher<br>Copyright Date<br>Cost | Learner Objective Covered | Comments |
|---|---|---|
| ALGEBRAIC EXPRESSIONS<br>• Operations with polynomials | | |
| • Operations with rational expressions | | |
| • Evaluating and solving algebraic expressions | | |
| LANGUAGE OF ALGEBRA<br>• Symbols and technical terms | | |
| • Properties of number systems | | |
| • Translating algebraic statements | | |
| TABULAR RELATIONSHIP<br>• Comparing tables | | |
| • Preparation and uses of tables | | |
| EQUATIONS<br>• Writing algebraic expressions | | |
| • Linear equations | | |
| • Word Problems | | |
| GRAPHS LINEAR FUNCTIONS<br>• Ordered pairs and points | | |
| • Graphing linear functions | | |
| • Interpreting graphs of linear functions | | |
| • Comparison of two linear functions | | |
| • Problem-solving by graphs | | |
| APPLICATIONS OF FORMULAS<br>• Applications | | |
| • Perimeter, area, volume | | |

| | Learner Objective Covered | Comments |
|---|---|---|
| NON-LINEAR FUNCTIONS<br>• Solving problems | | |
| • Graphs of $a = xy$ and $y = ax^2$ | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)
4. Does the student book have any special features? (e.g., enrichment activities)
5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?
7. Does the student text include a sufficient number of exercises?
8. Other comments:

CORE GEOMETRY

| Title<br>Author<br>Publisher<br>Copyright Date<br>Cost | Learner Objective Covered | Comments |
|---|---|---|
| GEOMETRIC FIGURES AND RELATIONSHIPS<br>• Angle relationships to solve problems | | |
| • Concepts relating to triangles | | |
| • Pythagorean theorem | | |
| • Notion of "congruent figures" | | |
| • Notion of "perpendicularity" to solve problems | | |
| • Conditions that imply and are implied by parallelism | | |
| • Properties of quadrilaterals | | |
| • Notion of "similar figures" | | |
| • Similarity problems | | |
| • Area and perimeter formulas of polygonal regions | | |
| • Circumference and area formulas of a circle | | |
| • Volume formulas of common geometric solids | | |
| TRANSFORMATIONS<br>• Use of translations, rotations, reflections, expansions, and contractions to demonstrate geometric concepts | | |
| • Use of symmetry to demonstrate geometric concepts | | |
| DEDUCTIVE SYSTEMS<br>• Organization into a deductive system | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (eg., enrichment activities)

5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

STATISTICS

| Title _____<br>Author _____<br>Publisher _____<br>Copyright Date _____<br>Cost _____ | Learner Objective Covered | Comments |
|---|---|---|
| • Descriptive and inferential statistics | | |
| • Construction of frequency distributions and other graphs | | |
| • Computation of percentiles and percentile ranks | | |
| • Calculations and comparisons of the mean, the median, and the mode | | |
| • Computation of range, average deviation, variance, and standard deviation | | |
| • Calculation of linear coefficients and drawing of scattergrams | | |
| • Use of the least squares regression equation and line | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:

   a. sequencing of topics?

   b. reading level?

   c. use of technical language?

2. Does the teachers' guide include:

   a. strategies for teaching?

   b. supplementary activities?

   c. suggested time schedule?

d. testing program?

e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:

a. problem solving strategies?

b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

SELECTED ALGEBRA TOPICS

| Title _____<br>Author _____<br>Publisher _____<br>Copyright Date _____<br>Cost _____ | Learner Objective Covered | Comments |
|---|---|---|
| COUNTING METHODS<br>• Fundamental counting principle | | |
| • Permutations | | |
| • Applications of permutation | | |
| • Combinations | | |
| SEQUENCES<br>• Investigation of sequences (e.g., arithmetic, binary, Fibonacci) | | |
| • Infinite sequences | | |
| FUNCTIONS AND THEIR GRAPHS<br>• Nature of a function | | |
| • Graphing and interpretation of linear or curved functions | | |
| MATHEMATICAL CURVES<br>• Circle, ellipse, parabola, and hyperbola | | |
| • Other curves (e.g., sine curve, spirals, cycloid, logarithmic curve) | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of;

   a. sequencing of topics?

   b. reading level?

   c. use of technical language?

2. Does the teachers' guide include:

   a. strategies for teaching?

   b. supplementary activities?

   c. suggested time schedule?

   d. testing program?

   e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:

   a. problem solving strategies?

   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

Title_____

Author_____

Publisher_____

Copyright Date_____

Cost_____

| | Learner Objective Covered | Comments |
|---|---|---|
| REAL NUMBERS | | |
| • Counting numbers, wholes, and integers | | |
| • Operations with integers | | |
| • Relationship between real numbers and decimals | | |
| • Number systems | | |
| • Graphing sets of real numbers | | |
| • Operations with rational numbers | | |
| • Properties of the real numbers | | |
| LANGUAGE OF ALGEBRA | | |
| • Nature and use of a variable | | |
| • Power and use of algebraic symbols | | |
| • Relationship between the properties of the real numbers and algebraic techniques | | |
| • Evaluating algebraic expressions | | |
| POLYNOMIALS | | |
| • Positive integral exponents | | |
| • Operations with polynomials | | |
| • Algebraic manipulation | | |
| LINEAR OPEN SENTENCES IN ONE VARIABLE | | |
| • Solving equations by trial and error | | |
| • Solution of equations of all types | | |
| • Solving equations and inequalities by formal methods; graphing solutions | | |
| • Solving formulas | | |
| • Solving problems by using open sentences | | |

| | Learner Objective Covered | Comments |
|---|---|---|
| LINEAR OPEN SENTENCES IN TWO VARIABLES<br>• Rectangular coordinate system | | |
| • Open sentence by trial and error | | |
| • Graphs linear equations and inequalities | | |
| • Finds equation from a graph | | |
| • System of linear equations | | |
| • Systems to solve problems | | |
| • Graphs system of linear inequalities | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?

2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g. enrichment activities)

5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

| Title<br>Author<br>Publisher<br>Copyright Date<br>Cost | Learner Objective Covered | Comments |
|---|---|---|
| PRODUCTS AND FACTORS<br>• Distributive law to find the product of binomials | | |
| • Special products: $(a \pm b)^2$; $(a + b)(a - b)$ | | |
| • Relationship between multiplication and factorization | | |
| • Factors polynomials | | |
| • Factoring to solve quadratic equations | | |
| RATIONAL EXPRESSIONS<br>• Operations with rational expressions | | |
| • Problems using fractional equations | | |
| RADICALS<br>• Rational approximations of radicals | | |
| • Radicals on the number line | | |
| • Operations with radicals | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:

   a. sequencing of topics?

   b. reading level?

   c. use of technical language?

2. Does the teachers' guide include:

   a. strategies for teaching?

   b. supplementary activities?

   c. suggested time schedule?

   d. testing program?

   e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:

   a. problem solving strategies?

   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

Title_____

Author_____

Publisher_____

Copyright Date_____

Cost_____

| | Learner Objective Covered | Comments |
|---|---|---|
| DEDUCTIVE AND INDUCTIVE REASONING<br>• Conjectures | | |
| • Nature of a theorem | | |
| • Undefined terms and development of definitions | | |
| • Definitions | | |
| GEOMETRIC FIGURES AND RELATIONSHIPS<br>• Angle relationships to solve problems | | |
| • Concepts relating to triangles | | |
| • Pythagorean theorem | | |
| • SS, SAS, ASA and HL triangle congruences | | |
| • Notion of "perpendicularity" to solve problems | | |
| • Conditions that imply and are implied by parallism | | |
| • Comparison of the properties of the quadrilaterals | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:

   a. sequencing of topics?

   b. reading level?

   c. use of technical language?

2. Does the teachers' guide include:

   a. strategies for teaching?

   b. supplementary activities?

   c. suggested time schedule?

   d. testing program?

   e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:

   a. problem solving strategies?

   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

Title _____

Author _____

Publisher _____

Copyright Date _____

Cost _____

| | Learner Objective Covered | Comments |
|---|---|---|
| GEOMETRIC FIGURES AND RELATIONSHIPS<br>• Formal definition of "similar figures" | | |
| • SSS, AA, SAS to show triangle similarity | | |
| • Similarity problems | | |
| • Area and perimeter formulas of polygonal regions | | |
| • Concepts relating to circles | | |
| • Circumference and area formulas of a circle | | |
| • Volume formulas of common geometric solids | | |
| TRANSFORMATIONS<br>• Translations, rotations, reflections, expansions, and contractions | | |
| • Combinations of transformations | | |
| • Notion of "symmetry" | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:

   a. sequencing of topics?

   b. reading level?

   c. use of technical language?

2. Does the teachers' guide include:

   a. strategies for teaching?

   b. supplementary activities?

   c. suggested time schedule?

   d. testing program?

   e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:

   a. problem solving strategies?

   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

ALGEBRA IIA

| Title _____<br>Author _____<br>Publisher _____<br>Copyright Date _____<br>Cost _____ | Learner Objective Covered | Comments |
|---|---|---|
| REAL NUMBERS:<br>• Definition of a real number | | |
| • Writing a repeating decimal as the quotient of two integers | | |
| • Relationship between the set of reals and its subsystems | | |
| EXPONENTS<br>• Laws of exponents (including rational exponents) | | |
| • Operations with radicals | | |
| FACTORING<br>• Techniques of factoring | | |
| • Application of factoring to rational expressions | | |
| RELATIONS AND FUNCTIONS<br>• Definitions of function and functional notations | | |
| • Graphing relations and functions | | |
| • Domain and range of a function | | |
| • Combination of functions | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:

   a. sequencing of topics?

   b. reading level?

   c. use of technical language?

2. Does the teachers' guide include:

   a. strategies for teaching?

   b. supplementary activities?

   c. suggested time schedule?

   d. testing program?

   e. other?

3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:

   a. problem solving strategies?

   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

Title _____

Author _____

Publisher _____

Copyright Date _____

Cost _____

| | Learner Objective Covered | Comments |
|---|---|---|
| POLYNOMIAL FUNCTIONS | | |
| • Graphing constant functions | | |
| • Graphing linear functions | | |
| • Solving linear equations and inequalities | | |
| • Solving systems of linear equations; graphing system of linear inequalities | | |
| • Graphing quadratic functions | | |
| • Solving quadratic equations and inequalities | | |
| • Solve problems using quadratic equations | | |
| • Graphing techniques | | |
| • Solving polynomial equations of degree more than two | | |
| • Graphing polynomial functions of degree more than two | | |
| RATIONAL FUNCTIONS | | |
| • Intercepts, symmetry, and asymptotes | | |
| • Graphing rational functions | | |
| INVERSE FUNCTIONS | | |
| • Notion of "inverse function" | | |
| • Finding $f^{-1}(x)$ from $f(x)$ | | |
| • Graphing $f^{-1}$ from the graph of a function f | | |
| • Graphing exponential and logarithmic functions | | |
| • Solving exponential and logarithmic equations | | |
| • Logarithmic tables | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any speical features? (e.g., enrichment activities)

5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

Title _____

Author _____

Publisher _____

Copyright Date _____

Cost _____

| | Learner Objective Covered | Comments |
|---|---|---|
| TRIANGLE TRIGONOMETRY | | |
| • Six trigonometric ratios | | |
| • Trigonometric tables | | |
| • Right triangle problems | | |
| • Cosine and sine laws | | |
| ANALYTIC TRIGONOMETRY | | |
| • Trigonometric functions | | |
| • Radian measure for angles | | |
| • Trigonometric values for the multiples of $\pi/6$ and $\pi/4$ | | |
| • Graphing periodic and bounded functions | | |
| • Graphing trigonometric functions | | |
| • Graphing $y = A \sin(Bx + C)$ and $y = A \cos(Bx + C)$ | | |
| • Trigonometric identities | | |
| • Addition and subtraction formulas, double-angle formulas, half-angle formulas | | |
| • Formulas for the sums and differences of sines and cosines | | |
| • Product formulas | | |
| • Trigonometric equations and inequalities | | |
| • Relation between trigonometric functions and their inverses | | |
| • Graphing the inverse trigonometric functions | | |
| • Equations involving the inverse trigonometric functions | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

ALGEBRA III

| Title<br>Author<br>Publisher<br>Copyright Date<br>Cost | Learner Objective Covered | Comments |
|---|---|---|
| LINEAR SENTENCES<br>• Equations and inequalities (one variable) | | |
| • Absolute value expressions | | |
| • Graphing equations and inequalities (two variables) | | |
| POLYNOMIALS AND FRACTIONAL EXPRESSIONS<br>• Algebraic manipulation | | |
| • Factoring and its applications | | |
| • Graphing polynomial equations | | |
| • Open sentences containing fractional expressions | | |
| SEQUENCES AND SERIES<br>• Notations and definitions | | |
| • Recursive definitions | | |
| • Arithmetic and geometric sequences | | |
| • Arithmetic and geometric series | | |
| • Limits of infinite geometric series | | |
| MATRICES AND DETERMINANTS<br>• Matrix addition and multiplication | | |
| • Determinants (include expansion by minors) | | |
| • Cramer's rule (3 x 3) | | |

| | Learner Objective Covered | Comments |
|---|---|---|
| REAL AND COMPLEX NUMBERS<br>• Relationships between the set of complex numbers and the reals | | |
| • Properties of equality and order | | |
| • Computations (include radicals, exponents, complex numbers) | | |
| • Equations with radicals | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)
4. Does the student book have any special features? (e.g., enrichment activities)
5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?
7. Does the student text include a sufficient number of exercises?
8. Other comments:

ANALYSIS

Title_____

Author_____

Publisher_____

Copyright Date_____

Cost_____

| | Learner Objective Covered | Comments |
|---|---|---|
| FUNCTIONS (algebraic, transcendental, discontinuous) | | |
| • Graphing | | |
| • Domain and range | | |
| • Inverse function | | |
| • Trigonometry (properties, identities) | | |
| • Trigonometry formulas | | |
| POINTS AND LINES | | |
| • Distance, midpoint formulas | | |
| • Equations of lines | | |
| $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ | | |
| • Standard forms of equations of conics | | |
| • Translation and rotation | | |
| • Graphing conics | | |
| • Determining equations of conics | | |
| POLAR COORDINATES | | |
| • Rectangular $\longleftrightarrow$ polar conversions | | |
| • Graphing polar equations | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?

6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

Title _____

Author _____

Publisher _____

Copyright Date _____

Cost _____

| | Learner Objective Covered | Comments |
|---|---|---|
| RECTANGULAR COORDINATES<br>• Distance and midpoint formulas | | |
| • Graphing equations and inequalities | | |
| • Writing the equation or inequality of a graph | | |
| $Ax + By + C = 0$<br>• Slope of a line | | |
| • Graphing lines | | |
| • Equations of lines | | |
| • Graphing linear inequalities | | |
| • Systems of linear equations | | |
| • Formula for distance from a point to a line | | |
| $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$<br>• Standard forms of equations of conics | | |
| • Translation and rotation of axes | | |
| • Completing the square | | |
| • Graphing the conic sections | | |
| • Graphing $Ax^2 + Bxy + Cy^2 + Dx + Ey + F \geq 0$ (or $\leq$) | | |
| • Determining equations of conics | | |
| • Relationship between $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and the conics | | |
| POLAR COORDINATES<br>• Polar coordinate system | | |
| • Rectangular $\longleftrightarrow$ polar conversions | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)

4. Does the student book have any special features? (e.g., enrichment activities)

5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?

7. Does the student text include a sufficient number of exercises?

8. Other comments:

| Title _____<br>Author _____<br>Publisher _____<br>Copyright Date _____<br>Cost _____ | Learner Objective Covered | Comments |
|---|---|---|
| **INEQUALITIES AND ABSOLUTE VALUES** | | |
| • Inequalities (including those with absolute value) | | |
| • Graphing functions with absolute value | | |
| **LIMITS** | | |
| • Definition of $\lim_{x \to a} f(x)$ | | |
| • Limit theorems | | |
| • Continuity | | |
| • Limits at infinity and infinite limits | | |
| **DIFFERENTIATION** | | |
| • Definition of derivative | | |
| • Derivatives using definition | | |
| • Theorems on derivatives | | |
| • Chain rule | | |
| • Differentiation of elementary functions | | |
| • Use of first and second derivatives in graphing | | |
| • Differentiation in solving problems | | |
| **INTEGRATION** | | |
| • Definite and indefinite integral | | |
| • Relationship between integration and differentiation | | |
| • Properties of the integral | | |
| • Approximation of definite integral | | |
| • Integration of elementary functions | | |
| • Use of appropriate techniques of integration | | |

| | Learner Objective Covered | | Comments |
|---|---|---|---|
| • Improper integrals | | | |
| • Integration in solving problems | | | |

OTHER CONSIDERATIONS:

1. Is the material appropriate in terms of:
   a. sequencing of topics?
   b. reading level?
   c. use of technical language?
2. Does the teachers' guide include:
   a. strategies for teaching?
   b. supplementary activities?
   c. suggested time schedule?
   d. testing program?
   e. other?
3. Are supplementary materials available? (e.g., tests, duplicating masters)
4. Does the student book have any special features? (e.g., enrichment activities)
5. Would the material help students to improve:
   a. problem solving strategies?
   b. communication skills?
6. Are practical applications included?
7. Does the student text include a sufficient number of exercises?
8. Other comments:

HIGH SCHOOL
SCHEDULE G
SUGGESTIONS FOR IMPLEMENTATION

The examples below provide suggestions for beginning the implementation of the department's improvement plan. These suggestions do not cover all the possibilities nor are all suggestions appropriate for every department.

1. contact District Mathematics Coordinator regarding
   - deadlines for in-service requests
   - in-service needs identified in the department's improvement plan
2. contact District Mathematics Coordinator regarding
   - availability of resource personnel (e.g., UH College of Education, community college, parent and community groups)
   - the nature and availability of District and State resources
   - the services of the District Mathematics Resource Teacher
   - examination copies of current textbook series, alternative mathematics programs, and supplementary materials
   - assistance in using or reviewing instructional materials in the District's Resource Center
3. contact textbook representatives and publishers through District Mathematics Coordinator regarding
   - availability of teacher in-service training
   - examination copies of instructional materials (e.g., for remedial, for enrichment, for incorporating procedures)
4. begin setting up a teacher resource room or area
   - collect examination copies of student texts and teacher references
   - inventory other instuctional materials (e.g., games, calculators) that are available in the department
   - subscribe to the Arithmetic Teacher and/or the Mathematics Teacher
5. schedule mathematics department meetings related to the improvement plan
   - schedule meetings for the first quarter
   - have a specific agenda for each meeting

6. plan for a quarterly newsletter highlighting accomplishments, current tasks, and future goals relative to the department's improvement plan

7. determine criteria for evaluating intervention strategies identified for improving the mathematics program

8. establish in-house in-service training for particular content areas and/or teaching strategies

9. plan for teachers to observe other teachers within the department

10. plan on visitations to other schools

11. contact schools within complex (elementary feeder schools, intermediate feeder schools, high schools) for articulation meetings

12. contact other high schools within the district to share information, concerns, expertise, and so on

# Trends and Issues in Mathematics Education

# 6. TRENDS AND ISSUES IN MATHEMATICS EDUCATION

## Introduction

Any issue in mathematics education is often thought of as two or more philosophies, recommendations, experimental projects, curricula, and so on, each seeking the same general goal. Any trend seems to be an agreement by factions with the differing viewpoints to work together for the common goal. That common goal, although expressed in various ways, is often "to improve mathematics education." Trends, in an attempt to move toward that goal, have taken various forms in the last twenty years. Some that are readily recognizable are:

- Emphasis on structure and unifying concepts
- Use of mathematics laboratories
- Preparing and implementing behavioral objectives
- Changing patterns of teacher education
- Equity in school mathematics opportunities

Of late two issues somewhat related, Back-to-Basics and Competency-Based Education, have attracted a great deal of attention. The first is evidenced by the publicity in newspapers, on television, in professional magazines, and the publication of textbooks with titles containing the word "basic" and a count of the huge number of computational exercises in the book is so often given to promote its worth. The competency-based education issue is a "hot" one. Nationwide, as of April 1977, at least twenty-eight states have moved or are moving to competency-based education. Hawaii is one of these. No resolution of either issue is in sight. Each is considered in more detail in the following pages.

Major Trends and Issues

Many lists of trends and issues of varying length have been compiled. The following classification is a composite of those published and those under discussion by teachers. This list like all others contains titles that involve more than one topic. There are important issues and trends that have not been included, not because they do not affect mathematics education, but because they involve the entire schools. Among these are discipline, involvement of teachers in policy decisions and budget.

1. Changes in Content of Mathematics Programs

There have not been major changes in the content of mathematics programs in the last ten years when compared to the period ten to twenty years ago. The recent changes have been changes in emphasis and changes resulting in delay in introducing topics, symbolism, and algorithms.

Some specific changes in content are the following:

a. Formal work with sets is deemphasized. Sets remain important but in early instruction, the words "group" and "set" are used interchangeably and the distinction is made in the upper grades as needed.

b. Bases other than ten are deemphasized or deleted.

c. Lessons on rational numbers (as equivalence classes) are delayed.

d. Symbolism such as $\cup$, $\cap$, $\in$, AB, $\forall$ and so on is delayed until really needed.

e. Algorithms such as those for division of fractions and long division with large divisors are delayed until grade seven or eight.

f. In algebra and geometry an appeal is made for intuition as well as for formal proof.

g. Instruction in the metric system of measurement as well as the customary system is provided.

It should be recognized that changes in content do not reject important goals of the "new mathematics". The goal of understanding procedures so often emphasized in the sixties remains a top priority. However, the changes are generally accepted by most mathematics teachers as being in the right direction.

There is one issue related to content that is being discussed and may attract more attention soon. Sometimes this has been referred to as the "content crunch" which describes the fact that during the sixties new topics were introduced, "traditional" topics were introduced at lower grade levels and the number of pages in books was increased. This remains in effect today. On the one hand, the need for more content in this technological age is recognized; but at the same time major increases in content at any grade causes more difficulties for more pupils.

2. Changes in Teaching Strategy

The revolution of the sixties was content oriented--the introduction of new content, the teaching of topics at lower grade levels, and so on and except for isolated instances there were no dramatic changes in teaching strategies. Many educators have said, "The New Math was taught in the same old way." Drill in "old math" was replaced by drill in "new math."

In the seventies some research on effective instructional strategies was conducted. In Hawaii helping teachers to use the language, experience

and thinking (LET) of the learner in effective instruction has been a concern.

Current national trends in teaching strategies are as follows:

a. Reduction in formalism

The discarding of extremes in detail, in symbolism, and in precise definitions is evident by examination of recently published books. The trend has been expressed in many ways. Two of these are:

- Pupils can grasp many complex ideas long before they can understand the word statement and symbolism describing it. (Think about commutativity for addition.)
- Informality and intuition are better teaching strategies than formal structures.

On the elementary school level ten years ago, teachers were urged to help pupils learn that three minus five is not impossible but cannot be named with a whole number. Now we hope pupils working with physical materials find they cannot remove five objects from a given set of three. In algebra precision of rules for adding integers covering half a page of difficult reading with complex symbolism was in vogue a decade ago. Now we tend to emphasize addition of integers as counting on and securing answers intuitively, with formal rules delayed until more experience with absolute value and inequalities is accumulated.

b. Increased emphasis on concepts

Presently, attention is being focused on experiences

which help students continuously refine the meaning of mathematical ideas. For example, students' introduction to $\pi$ is through measuring diameters and circumferences of circles and comparing by division; later they find various approximations to $\pi$; still later they find many applications and one day learn it cannot be expressed as a fraction.

We may not have a theory that describes exactly how concepts are learned but we do know that they are developed and are refined over a long period of time through a sequence of experiences. Further, we know their extreme importance in learning computational procedures and learning to solve problems.

The importance of concepts is identified at various places in the Mathematics Program Guide: on page 27 in the "Scope and Sequence Chart"; on pages 39-156 where "Learner Objectives and Related Comments" contain explicit and implicit reference to concepts.

c. The place of drill

Drill is important today. The issues are: what is the purpose of drill, how can it be meaningful and efficient, and how does it relate to other instructional activities. Today the teacher must search for a variety of drill exercises and for some way of determining the amount of drill necessary for various students, find a plan of spaced practice for maintenance of skills, make decisions on exactly what content is to be learned as a skill by drill, and determine what is

the proper balance between drill and other kinds of learning within his/her class.

d. Emphasis on applications and problem solving

The emphasis on relevant applications and problem solving is to some extent both a trend and an issue. The importance of applications is definitely recognized and the ability to solve problems is regarded by most mathematics educators as an ultimate goal of the curriculum. However, for too many teachers, applications mean word problems in textbooks, where the major task of the pupil is to translate the technical jargon into simpler language and then into suitable symbolic form.

In its list of recommendations for school mathematics of the 1980's, the National Council of Teachers of Mathematics recommended that problem solving be the focus of school mathematics. It stated that the mathematics curriculum should be organized around problem solving and that mathematics programs should involve students in problem solving by representing applications at all grade levels.

The NACOME report states, "Applications include all of the practical uses of mathematics needed by everyone for daily life today and in the future, a need...poorly fulfilled by present...programs. But applications in school mathematics cannot be limited to this important dimension. They must include all problems susceptible to mathematical analysis..."

So despite the valid need for more applications, there seems to be a lack of them. In addition there are many

questions and problems concerning applications:

- Which concepts or skills are taught most effectively through specific real life models?
- Are applications best used to introduce or to illustrate mathematical ideas?
- Does the use of applications improve interests and attitudes of students?
- Where can the teacher find help in securing these applications?
- Are applications best taught through an interdisciplinary approach?

The importance of problem solving is highlighted in the Mathematics Program Guide for the State of Hawaii on the following pages: page 13, as one of four broad goals for the mathematics program; pages 16-17, where experiences to promote problem solving abilities are listed; pages 26-27, where problem solving is found in the program for every grade K-12.

e. The place of discovery

This is an issue with opponents arguing that, as a method, discovery takes too long with uncertain results and proponents citing the thrill of discovery and as a consequence improved attitudes and interest of students. The method, illustrated by the lessons of Max Beberman, Robert Davis, David Page, and others, is a thing of beauty and effectiveness when used for appropriate topics by the competent teacher who is well prepared and who believes in the strategy of

discovery. But it is a fiasco when used on an inappropriate topic or when attempted by a good teacher with no faith in its worth.

Certainly discovery is not to be used every day nor for every topic. It is only one instructional strategy that the teacher should know and use with judgment. No one can prescribe which topics lend themselves to the discovery approach. Further, we do not even know if it is most effective for students of certain ability or maturity levels. In addition, the literature is filled with variations of discovery such as guided discovery, inductive discovery, deductive discovery, and so on. The teacher should be knowledgeable about discovery and prepared to make judgments and adaptations on its use.

It seems that the best advice to teachers is first examine the objectives carefully. If the teacher understands discovery then he/she should decide if discovery may be the instructional strategy to accomplish that objective.

The method of discovery is neither recommended nor rejected as a teaching strategy. However, many learner objectives and related comments in the Mathematics Program Guide use words such as investigate, explore, experiment, organize, interpret, devise, guess, and so on. When students engage in such activities under needed guidance from the teacher, discovery of ideas is one possible outcome.

f. The place of mathematics laboratories and the use of physical materials

There is great published enthusiasm for mathematics laboratories and the use of manipulatives, K-12. It is emphasized here that the interaction of students with materials form the basis of learning, i.e., students learn by doing. Yet, except for the primary grades, extensive use of concrete representations of mathematical ideas has never been popular. Seemingly, more and more, purchased materials are being stored in teachers' closets. One reasons seems to be the discipline problems created with laboratory lessons. Another seems to be the planning involved in using the materials.

Research evidence tends to support patterns of instruction that contain carefuly planned experiences with physical materials. A very serious mistake with mathematical laboratories or physical materials is to expect serendipity to replace planning. More planning is required. Contentions that learning outcomes in such programs are fragmented and difficult to assess or even based on faith are probably due to the lack of carefully planned experience.

A case can be made for experiences with materials and pictures at all grades. From a beginning concept, "What is two" to one encountered by only a few students at grade 12, "What is meant by a derivative," extremely effective lessons involving physical or pictorial experiences can be identified.

There are, then, advantages and disadvantages to the use of physical materials. It would be futile to try to decide in general which list is stronger. Decision "to use" or "not to use" should be made at the classroom level with decision based on a number of criteria ranging from the equipment of

the classroom, while considering the objective of the lesson, to the ability of the student. For this reason, the Mathematics Program Guide, while recommending strongly the need for students to represent ideas physically, does not provide a list of objectives best learned in a laboratory situation.

3. Hand-held calculators in the classroom

Answers of "Yes" and "No" both remain strong in answer to the question of "Should hand-held calculators be used in the classroom?" Arguments, written and oral, come not only from school personnel at all levels of instruction but also from parents, manufacturers, professional organizations, and so on. The general consensus seems to be that the hand-held calculator cannot be ignored. Yet nearly every knowledgeable person in this field believes research is needed to further explore its use.

A National Council of Teachers of Mathematics position statement (September 1978) still summarizes the present status of the issue:

> The National Council of Teachers of Mathematics encourages the use of calculators in the classroom as instructional aids and computational tools. Calculators give mathematics educators new opportunities to help their students learn mathematics and solve contemporary problems. The use of calculators, however, will not replace the necessity for learning computational skills.

As instructional aids, calculators can support the development and discovery of mathematical concepts. As computational tools, they reduce the time needed to solve problems, thereby allowing the considerations of a wider variety of applications. Furthermore the use of calculators requires students to focus on the analysis of problems and the selection of appropriate operations. The effective use of calculators can improve student attitudes towards, and increase interest in mathematics.

Other electronic devices, programmed to generate questions and activities, that provide immediate feedback to students, are not to be confused with calculators or computers. These devices can be used to reinforce computational skills through drill.

Furthermore, the National Council of Teachers of Mathematics recommended that school mathematics programs of the 1980's take full advantage of the power of the calculators at all grade levels.

4. Classroom organization

The many different classroom organizations proposed have one goal although expressed with different words. The curriculum must be planned so:

- It is adapted for the individual student.
- Each student progresses to the limit of his/her potential.

The variety of organizations include individualization, small groups, a single group, team teaching, and so on,

with numerous variations of each. For each of these plans there is a range of packaged materials (further subdivided for slow learners, fast learners, and so on) to teacher-made schemes which in some cases are daily ditto master worksheets.

There are obviously advantages and limitations for each plan. The advantages identified by satisfied users of any plan include the following:

- It facilitates class management.
- It promotes active involvement of the learner.
- It promotes interaction among pupils and among teacher and pupils.

It is interesting that the same advantages are stressed by programs whose plans seem to conflict.

Limitations of many plans seem to center on the complexity of the bookkeeping required, the cost for equipment and the fact that teachers are not prepared for the intricacies of the plan. Of course for most plans there are always statements that materials to support the plan are inadequate and the cost is too great. Claims of a panacea for any plan are exaggerations.

The type of classroom organization is ordinarily not mandated at the state or district level. Such is the case in Hawaii where nearly every teacher has a number of options from which to choose. Teachers should adapt organizational strategies after careful evaluation of the advantages and disadvantages, considering what will work best for the particular teaching situation.

5. Back-to-Basics

The "Back-to-Basics" movement has a long history and can be identified by many different titles. Over this period different groups have supplied leadership with varying philosophies and varying impact on education. The latest emphasis on back-to-basics has been supported by parents and legislators who believe the schools, while spending more money, have not accomplished their purpose. They cite illiterate high school graduates, inflated grading policies, courses with no basis in any discipline, and declining standardized test scores as evidence.

The national assessment results for the 1977-78 mathematics assessment indicate continued strength in computational skills with whole numbers and decline in problem-solving skills. This reflects narrowing of the mathematics curriculum in response to public support of back-to-basics--a narrowing that is an overreaction. This narrowing is evidenced by the following:

- Textbook publishers advertise "basics" in books that have much more computational material and less conceptual and problem material.
- Teachers, in general, are providing more drill.
- Programs, advertised as "individualized" in which pupils so often practice only computation, are popular.
- Proficiency testing is increasing.

The Mathematics Program Guide does not refer to "Back-to-Basics" explicitly. It recognizes the importance of basic

skills with the first reference on page 2 and continues through learner objectives at all grades. The Department of Education's Foundation Program Objective I is concerned with basic skills; and the Performance Expectations for grades K-3, six, eight, ten, and twelve (Mathematics Program Guide, pages 30-31) summarize the basic skills.

The National Council of Supervisors of Mathematics position paper on basic skills was an explicit attempt to focus the positive direction of back-to-basics for mathematics programs. (See Appendix B of the Mathematics Program Guide.)

The National Council of Teachers of Mathematics has recommended that "basic skills in mathematics be defined to encompass more than computational facility" in its list of recommendations for school mathematics of the 1980's.

6. Competency-based education and minimum competency

This is one of the "hottest" issues, not only in Hawaii, but also nationally. It is closely related to Back-to-Basics and evaluation which determines effectiveness of programs as well as competence of students.

Competency-based programs came into focus in the recent past as educators wrote long lists of behavioral objectives. These were the basics for experiences in the classroom and then tests were produced to evaluate accomplishment of the objectives. Regardless of criticisms that the objectives were of lower order mental abilities, such as skills in operations, and omitted the higher order abilities, such as reasoning, there have been great gains for many segments

of the teaching profession. Any time teachers think carefully and record the goals of their instruction, they are better prepared to provide an effective program.

The debate over minimum competency remains in the shouting stage with emotions taking the place of logic because evidence is meager to support either side of the issue. Some school districts or states which have adopted minimum competency report greater achievement, more interest and better attendance by pupils. Other districts and even states have postponed implementation of minimum competency procedures even after adopting and funding them.

There is no resolution in sight for this issue. Many questions requiring thought and research have been raised. Among them are:

- What are the minimum competencies in mathematics?
- How are they different for various individuals?
- How high are they set?
- Will establishing them cause a lowering of standards?

Hawaii is moving to competency-based education. The Hawaii State Department of Education has developed performance expectations in twenty-one areas, including mathematics, for grades three, six, eight, ten and twelve. It has also identified fifteen essential competencies for high school graduation. Mathematics teachers must thoroughly understand the performance expectations in planning programs and instruction.

7. Communication of Mathematical Ideas

That "word problems are difficult" has been common knowledge for years. It is especially true in Hawaii with scores of

pupils approaching the national average in computation on normed tests and scores on word problems definitely below the national average. This statistic describes the situation not only for the present but also for the past number of years.

If we were to ask ourselves or fellow teachers,"What have I (or you) done to improve pupils' ability to solve word problems?", a truthful answer would be "not much". If we pooled our ideas we would probably have fewer strategies than needed to do an effective job. This is a field that needs much study and research.

In developing skills for writing, reporting, or listening to mathematical ideas we have also done less than needed for literacy. For example, some questions on the recent National Assessment of Educational Progress tests were read to examinees. Listening and interpreting skills were reported as less than adequate.

As teachers our first task is to convince ourselves of the importance of these communication skills. Then we must incorporate the necessary objectives into the curriculum with the needed experiences to accomplish those objectives. The needed experiences included pupils reading and reporting in their own words, describing orally or in writing the meaning of a symbol or expression, drawing a picture of a numerical situation, rewording definitions or procedures, using systematic vocabulary, and so on.

The importance of communication of mathematical ideas is recognized in the Mathematics Program Guide in the

following places: on page 2 as one of the emphases of the guide; on page 30 where "Develop basic skills for learning and effective communication with others" is Foundation Program Objective I; on pages 39-156 where many Learner Objectives and Comments are directly related to communication skills.

8. Evaluation

The entire process of evaluation is often overwhelming for teachers. They find a variety of tests: achievement, diagnostic, inventory, attitude, and so on. They are told that evaluation in their classroom goes beyond paper and pencil tests, to include information from interviews, observations, rating scales, and so on. They are expected to know norm-referenced and criterion-referenced tests along with varieties of each. Then, with test results come the technicalities of item analysis, deviations, correlations, and so on. At some level of the school's administration, the technical aspects of evaluation are extremely important. Teachers are concerned with tests that will determine progress of students toward pre-determined goals.

One aspect of evaluation that is so often ignored at the classroom level is the establishing of objectives for a course, a chapter, or a lesson. Many times in preparing a test we look in a book and pick out exercises like those studied with little thought to "Do these really test the objectives?" This is one area that teachers can improve their evaluation techniques.

At the same time there are many practical questions for which teachers need answers:

- How can I use the results of the administration's evaluation program to improve my program?
- What (if any) are alternatives to paper and pencil tests?
- What are some suggestions for writing test questions?
- How can I evaluate higher order mental skills or even problem solving strategies?

One evaluation so seldom made is of the mathematics program for a grade or school. This can be carried out as a self-study by the department (secondary), a group of teachers (algebra or fourth grade), or even a single teacher. Searching questions concerning objectives, experiences of pupils, worth of texts, physical equipment in the classroom, and so on lead to a better understanding of any program. Even a comparison of the present program to that in the Mathematics Program Guide provides data for needed improvement, if any.

9. Equity in school mathematics opportunities

Truly equal opportunity to achieve in school mathematics for various groups of students has been lacking. This lack has become an issue of national concern. Groups such as girls, and culturally different or exceptional students have been the topics of debate. The passage of Public Law 94-142 in 1975 reflects a major commitment by this country to educate all handicapped children. Mainstreaming, that is, providing the most appropriate education for each child in

the least restrictive setting has become a challenge to the mathematics classroom teacher. Other exceptional students, namely the talented and gifted, are receiving increased attention.

In its recommendations for school mathematics of the 1980's the National Council of Teachers of Mathematics has recommended that "a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population." In its newly designed secondary mathematics program in the Mathematics Program Guide, Hawaii has made provisions for increased educational opportunities in mathematics.

The low rate of participation of girls in the study of mathematics has become a concern not only to the traditional feminists who have raised many questions about sex stereotyping but also to members of the traditional education community such as the National Institute of Education, Office of Education and various schools and professional associations across the nation. The National Council of Teachers of Mathematics has also adopted a position statement stressing the need for equality of opportunity for girls and young women in mathematics study. The low rate of participation in the study of mathematics in high school has closed many doors in both employment and continuing academic progress.

In 1968 and in 1974 Congress established the right of every non-English-speaking child to a bilingual program of education. The implications of such a program in mathematics

are enormous.

For the handicapped student, the individual teacher chooses the program which fits the student's specific skill needs. If the student's skill needs are in mathematics, then the teacher can request an evaluation of the student for special education services. When a student in special education is ready to be mainstreamed in a regular class for mathematics, the special education teacher usually attempts to place the student in the same text as is being utilized in the regular class to which he/she will be mainstreamed. This method is utilized to ensure consistency and success in the regular classroom.

Opportunities available in mathematics for the academically gifted and talented student include 1) enrichment activities at the elementary and intermediate levels; 2) mathematics contests; 3) Advanced Placement (AP) courses; and 4) early enrollment at the University of Hawaii and community colleges. Enrichment activities generally involve more in-depth study of mathematics concepts, problem-solving, other topics in mathematics that may not normally be covered in a regular program during the school year. Mathematics contests, sponsored by the Hawaii Council of Teachers of Mathematics (HCTM), are held each year throughout the state. By the competitive nature of these contests, they are limited to the most capable students. Advanced Placement courses in calculus are offered at various schools. Although some of the students who take the AP courses are not necessarily academically gifted, they are

usually the brightest mathematics students at those schools. Lastly, early enrollment at the University of Hawaii, or the community colleges is available for students who wish to enroll in advanced level courses.

Developing mathematical programs to meet the variety of interests, abilities, and goals of exceptional students places a special responsibility on mathematics educators. In An Agenda for Action: Recommendations for School Mathematics of the 1980s the National Council of Teachers of Mathematics makes the following statement[1]:

> Recognizing diversified individual interests and needs entails devising programs that are tailored for particular categories of students. Differentiated curricula must incorporate the special needs in mathematics of students with handicaps, including physical or learning difficulties. These programs will need to move away from the idea that everyone must learn the same skills...
>
> In many current programs, the student who does poorly in the algorithmic skills finds progress in all aspects of mathematical development halted, since remediation is designed to concentrate solely on this deficiency. Remedial programs should identify other areas of mathematical ability--for example, spatially related skills--and concentrate attention also on the students' strengths, not solely on their deficits.
>
> The student most neglected, in terms of realizing full potential, is the gifted student of mathematics.

[1] National Council for Teachers of Mathematics. An Agenda for Action: Recommendations for School Mathematics of the 1980s. Reston, VA: the Council, 1980.

## 7. PROBLEM SOLVING

Problem solving is a critical goal of the mathematics curriculum. The development of problem-solving skills is one of the eight Foundation Program Objectives adopted by the Department of Education. As such, problem solving must be considered one of the basic skills that each student must develop.

Mathematical problem solving is often thought of as synonymous with solving verbal textbook problems. However, solving story problems is but a small part of the skills that a student needs to develop in the problem-solving process.

Mathematical problem solving, in a broader context, refers to finding an appropriate response to a new situation or to a question which does not have an obvious answer; that is, getting the answer involves more than an act of simple memory or the application of a familiar algorithm. The problem solver must analyze the situation and determine the best strategy among a choice of many.

Often, getting the correct answer to word problems in textbooks is a mere exercise involving the application of a previously learned algorithm. Instructional practices that overemphasize algorithms and exercises do not encourage students to analyze and think through "story problems," nor do they build a foundation for developing skills with which to approach unfamiliar problems.

Problem-solving skills cannot be developed through occasional exposure to problems, but need to be a part of the daily classroom experience. The teacher plays a key role in directing and guiding the development of problem-solving skills, and in establishing a climate for promoting problem-solving behavior. A problem-solving approach to mathematics instruction fosters student involvement in the learning

process through questioning and cueing, providing the time to think through and analyze the situation, and emphasizing the process or strategies involved, rather than getting the "correct" answer. Students should be encouraged to question, explore and suggest alternatives in a problem situation.

Problem solving involves a number of interrelated skills and processes and cannot be taught through recipes and formulas. Instruction will need to provide opportunities for students to engage in problem solving to develop their problem-solving abilities. Instructional emphases of a problem-solving approach in mathematics are discussed below:

1. Problem-Solving Heuristics

   While problem solving cannot be reduced to a formula or an algorithm, there are a variety of strategies that can be taught to improve students' problem-solving skills. These strategies do not guarantee successful problem solving but serve as a guide or rule of thumb.

Most of the materials on problem solving follow the four stages of the problem-solving process identified by George Polya. The table below reflects these four stages and describes behavior of students.

| PROBLEM-SOLVING STAGES |
| --- |
| **Understanding the Problem**<br>• Asks questions to clarify problem<br>• States problem in own words<br>• Determines relevant facts in the problem<br>• Perceives implied relationships |
| **Devising a Plan**<br>• Summarizes data by making a table, graph or diagram<br>• Analyzes information (e.g. looks for patterns, works problem backwards, makes predictions and verifies, decomposes problem into parts)<br>• Recalls related problems previously solved<br>• Estimates solution |
| **Carrying Out the Plan**<br>• Uses a table or diagram to arrive at solution<br>• Applies a formula<br>• Performs computation required for solution<br>• Decides on where to begin<br>• Switches strategy when it is no longer applicable |
| **Looking Back**<br>• Describes strategy used in solving the problem<br>• Verifies that solution satisfies conditions of the problem<br>• Looks for alternative ways to solve the problem<br>• Creates applications or related story problems |

The role of the teacher is to facilitate this process by modeling these behaviors, and questioning or cueing students during their problem-solving experiences. The emphasis of problem-solving instruction should be on the process or strategies and not on getting the "correct" answer. Specific instructional behaviors may include:

- asking leading questions instead of giving answers.
- rewarding a good question, a different approach or an alternative procedure, even if it does not lead to the "answer".
- providing enough time for students to think about the problem (if the "answer" is obvious, then there was no problem).

2. Communication Skills

Discussion is an important aspect of problem solving. Students should be provided many opportunities for developing language and communication skills in the problem-solving process. These include:

a. asking questions for clarification
b. describing strategies used to solve a problem
c. listening to alternative ways of solving a problem
d. discussing the relative merits of alternative methods
e. sharing insights in a problem situation
f. writing extensions or related problems

The teacher-student and student-student interactions provide diagnostic information which can be used to determine appropriate instructional emphases. Language is critical in the process of connecting abstract symbols and concepts to referents in the student's experience, and in describing patterns and relationships the student perceives. These language experiences provide insights into student thinking.

3. Applications to Student Environment

The National Council of Supervisors of Mathematics has identified applying mathematics to everyday situations as one of the ten basic

skill areas. These include providing students with experiences in applying mathematics to all practical uses in daily life, in social and natural sciences, and in consumer and career related areas. Opportunities for students to relate mathematical skills and concepts to the real world promote the development of functional competence. For instance, students need to learn both how to compute and when to compute. The results of a national assesment show that "...skills are not mastered at the time when greatest emphasis is given in the curriculum but at a later date after practice and application."*

These results suggest that the instructional practice of teaching for mastery of skills before introducing applications may be depriving some students of the very experiences that would lead to mastery of these skills.

4. Use of Manipulatives and Representations

Mathematics concept development should proceed from the manipulative or concrete level to the representational or pictoral to the abstract or symbolic. Exploratory activities with manipulatives provide students with the experiences necessary for developing concepts, and acquiring vocabulary and appropriate language to describe the objects they are manipulating.

The transition from the manipulative to the representational level varies with each student. Language plays a critical role in connecting these levels. As the level of abstraction increases from manipulating objects to manipulating pictures and symbols, language serves as the bridge that connects the symbols to their referents in the real world and provides the context for meaningful learning.

*Carpenter, et. al. Results from the Second Mathematics Assessment of the National Assessment of Educational Programs. Reston, Virginia: National Council of Teachers of Mathematics, 1981, p. 13.

5. Independent Investigations

Every problem may be extended or modified to create a new, related problem. Given variations could result from changing the given context, numbers, or conditions. Initially these changes and extensions may be suggested by the teacher, but eventually students should be encouraged to investigate extensions or variations on their own. For example:

> Given the following problem:
>
> There were 8 people at a meeting. Every person shook hands with each other person exactly once. How many handshakes were there?
>
> Possible variations and extensions include:
>
> a. changing the number of people to 12
> b. changing the given to 28 handshakes and asking how many people were at the meeting
> c. generalizing to any number, (n) of people

6. Computational Skills

One of the ten basic skill areas of mathematics identified by the National Council of Supervisors of Mathematics is appropriate computational skills. While the availability of calculators minimizes the need for long and complicated computations, knowledge of single-digit number facts, mental arithmetic and estimation skills are still essential.

The Department of Education has identified fifteen essential competencies considered to be the minimum required for becoming productive and contributing members of society. One of these is related to computation.[1]

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[1]Department of Education, State of Hawaii, Office of Instructional Services. Student Performance Expectations of the Foundation Program. August, 1978, p. 107.

> Use computational skills in situations common to everyday life. These include adding, subtracting, multiplying, and dividing whole numbers, adding and subtracting dollars and cents, and computing discount and simple interest.

The statement of this essential competency focuses on using computational skills. Research in mathematics education has found that computational skills learned in isolation at a rote, mechanical level contribute little to the ability to use mathematics in everyday life.

Instructional strategies for developing and maintaining computational skills must be meaningful to the learner. These strategies include using manipulatives for concept development and applying skills to the student's daily experiences that were discussed above.

The instructional emphases discussed above are interrelated and overlap with each other. Each supports the development of the ability to reason and think through unfamiliar situations. Problem-solving skills are not learned in a short unit; rather, the learning of problem solving skills is a continuing process of development and refinement.

Teaching problem solving is more than a collection of instructional techniques. It is an attitutde and an approach to teaching in an environment that fosters thinking about problems, and encourages and values alternatives and creative insights.

## Annotated Bibliography


Department of Education, State of Hawaii, Office of Instructional Services. Problem Solving. 1980.
A resource guide to provide teachers with guidelines and materials in order to structure a course that would teach students mathematical content while learning problem-solving techniques. The course is an Option X, level B course in the restructured, grades 9-12, mathematics program.

Krulik, S. and Rudnick, J.A. Problem Solving: A Handbook for Teachers. Boston: Allyn and Bacon, Inc., 1980.
Includes an introduction to problem solving with a workable set of heuristics and the pedagogy of problem solving with discussions, problems, and activities for all grade levels.

Krulik, S. and Reys, R. (eds.). Problem Solving in School Mathematics: 1980 Yearbook. Reston, Virginia: National Council of Teachers of Mathematics, 1980.
Contains something to help every teacher of mathematics teach problem solving. Ideas at all levels of instruction are included. Here are problems, examples, and illustrations.

National Council of Teachers of Mathematics. An Agenda for Action: Recommendations for School Mathematics of the 1980's. Reston, Virginia: NCTM, 1980.
The National Council of Teachers of Mathematics' eight recommendations for school mathematics of the 1980's are made. It recommends that problem solving be the focus of school mathematics. Throughout the document the relationship of problem solving to the other recommendations is discussed.

National Council of Teachers of Mathematics. Arithmetic Teacher. November 1977.
This entire issue is devoted to summaries, discussion, and ideas of problem solving.

National Council of Teachers of Mathematics. Arithmetic Teacher. February 1982.
The issue focuses on "Teaching Problem Solving." Articles span primary to junior high school levels and contain ideas and suggestions for engaging students in problem-solving experiences.

Polya, G. How to Solve It. Princeton, New Jersey: Princeton University Press, 1957.
A classic and a must for mathematics teachers. The stages and behaviors of problem solving as discussed in the Mathematics Program Guide are discussed further here.

## 8. READING IN MATHEMATICS

The right to read includes the right to read mathematics. According to the State Reading Improvement Framework[1], the content area of mathematics should include strategies for reading mathematics. To assist teachers in carrying out their responsibility to teach reading in mathematics, the Department of Education has made available several documents. These are:

Language Arts Strategies for Basic Skills, K-2. Honolulu: Department of Education, State of Hawaii, 1978.

Comprehension in the Content Areas 3-6, Strategies for Basic Skills. Honolulu: Department of Education, State of Hawaii, 1979.

Comprehension in the Content Areas 7-12, Strategies for Basic Skills. Honolulu: Department of Education, State of Hawaii, 1979.

These materials offer teachers and administrators suggestions on how to assess their students' reading comprehension level and on analyzing and selecting instructional strategies and activities. In addition to the general reading references made available by the Department of Education there are references pertaining specifically to the reading of mathematics. These are provided in the annotated bibliographic listing. Course work on reading in content areas and specifically in mathematics is provided by the College of Education of the University of Hawaii at Manoa. General reading resource personnel are also available at each of the districts.

To assist teachers and schools in assessing the reading levels of instructional materials in use or intended for use by the school several assessment tools are discussed in this section. These assessment tools are basically of two types: those that actually involve students reading the materials being assessed and those in which the materials are assessed without student involvement.

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[1]Department of Education, State of Hawaii, Office of Instructional Services. State Reading Improvement Framework. June 1979.

The Cloze Technique. This technique involves the students reading passages from the reading materials being assessed. What follows are instructions for its construction, administration, scoring, and interpretation.

Construction:

1. Select a reading passage of approximately 275 words from material that you will be assigning to your students. This should be material that they have not yet read.
2. Type the selected passage. Leave the first sentence of your selection intact. Starting with the second sentence, select at random one of the first five words. As you type the passage, leave an underlined blank 15 spaces long to replace the word you have chosen. A shorter blank, 5 spaces long, should be used for mathematical symbols (or tokens) where the context demands it.
3. Delete every fifth word or mathematical token and replace each with a blank thereafter, until you have a total of 50 underlined blanks. Finish that sentence which includes the fiftieth blank.
4. Type one more sentence intact.

Administration: The cloze test can be easily administered by individual teachers within their own classrooms through the following directions:

1. Students are not to use their textbooks in completing the cloze exercise.
2. Before passing out the tests, inform students that they will be taking a test that will try to measure the difficulty of their class reading materials. Show them how the cloze works on the board with sample sentences such as, "It's dark in here. Please turn on the _____." Or, "The man _____ down

the stairs." Emphasize to students that they can get clues from the context of the reading passage that will help them determine words that fit.

3. Allow students the entire class period to complete the test. (Many should finish early.)

Scoring:

1. Count as correct every exact word students supply. Do not count synonyms judged to be satisfactory. Counting synonyms will not improve the usefulness of the test; the rank order performance of the class will be essentially unchanged and it is sometimes extremely difficult to judge the correctness of a synonym.
2. Multiply the total number of exact word replacements by two in order to determine the student's cloze percentage score.

Interpretation:

The results of this assessment are for class interpretation, not individual assessment. Hence a profile of class results is needed for interpretation.

Generally speaking scores below 40% indicate a difficult reading, scores between 40% and 60% indicate a satisfactory reading, and scores above 60% indicate an easy reading level. To interpret results, the number or percent of students functioning at each of these three levels must be viewed. Instructional materials are best, from a reading-level viewpoint, if the majority of the students find the materials at a satisfactory reading level.

An example of a cloze test follows.

Number of Bananas

Skip, 3 + 2 _____ 5

Chip, 2 + _____ = 5

So 3 _____ 2 = 2 + _____

Changing the order of _____ addends does not change _____ sum.

This is the _____ property of addition.

1. _____ the sums. Do the _____ in the parentheses first.

   _____. (5 + 3 _____ + 4; 7 + _____ 3 + 4)

   _____. Are the sums the _____?

   (8 + 7 _____ + 6 = 2_____ and 8 + ( _____ + 6) = _____1.

   so (8 _____ 7) + 6 _____ 8 + (7 _____ 6)

Changing the _____ of the addends does _____ change the sum.

This _____ the grouping property of _____.

2. Complete

   a. 4 _____ 3 = 7, so _____ + 4 = ☐.

   _____. (4 + 2 _____ + 6 = 1_____, so 4 + ( _____ + 6) = ☐.

____ . Add.

   a. 8 + _____ b. 0 + 8 _____. 9 + 0

   7 _____ 0 = 7 98 + _____ = 98

When _____ add 0 to a _____, the sum is that _____.

This is the property _____ zero for addition.

4. _____ properties are shown?

   a. _____32 + 0 _____ 632 b. _____3 + 42 _____ 42 + 2_____

   c. (12_____ + 354 ____ + 145 = _____23 + (_____54 + 145)

5. Make true sentences.

   a. 34 + 0 = ☐ b. 12 + 2 = 2 + ☐

From Holt School Mathematics, Grade 4 by Eugene D. Nichols, et al. New York: Holt, Rinehart & Winston, 1978, pp. 36-37. NOTE: Key to this test on p. 7-5.

CLOZE TEST KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | = | 11. | ) | 21. | + | 31. | 2 | 41. | What |
| 2. | 3 | 12. | ( | 22. | grouping | 32. | 3 | 42. | 6 |
| 3. | + | 13. | b | 23. | not | 33. | 0 | 43. | = |
| 4. | 3 | 14. | same | 24. | is | 34. | c | 44. | 2 |
| 5. | the | 15. | ) | 25. | addition | 35. | + | 45. | = |
| 6. | the | 16. | 1 | 26. | + | 36. | 0 | 46. | 3 |
| 7. | order | 17. | 7 | 27. | 3 | 37. | we | 47. | 3 |
| 8. | Find | 18. | 2 | 28. | b | 38. | number | 48. | ) |
| 9. | work | 19. | + | 29. | ) | 39. | number | 49. | 1 |
| 10. | a | 20. | = | 30. | 2 | 40. | of | 50. | 3 |

The Fog Index. This technique does not involve students in the assessment process. Note that the technique assesses readability by determining the length of words and sentences in sample passages of the book or unit. What follows are instructions in the use of the Fog Index.

1. Take three 100-word passages; one from the beginning, one from the middle and one from the end of the chapter or book.
2. Count the number of words in each passage that have three or more syllables. Note this down for use in step 4. Do not count proper names, compound words or verb forms that become three syllables by adding suffixes. If the same three-syllable word appears several times, count it on each occurrence.
3. Determine the average sentence length for each passage by:
   a. Totaling the number of words in the passage. Include the words in the sentence containing the hundredth word in your count.
   b. Totaling the number of complete sentences in the selection. In the total include decimal fractions of partial sentences. For a partial sentence, estimate the percentage that is included in the 100-word passage, for instance, .6 of a sentence.

c. Divide the number found in a. by the number found in b.

4. Total factors from steps 2. and 3.c. (the number of three or more syllable words and the average sentence length), and multiply the sum by 0.4. The result is the Fog Index for that passage. The score (Fog Index) represents the approximate level of education needed to read the passage.

The determination of level of difficulty of the first passage chosen from a text whose estimate of readability is desired is shown on the following page.

To estimate the readability level of this text, estimates of level of difficulty must also be made of passages from the middle and from the end of the book. Estimates of level of difficulty of passages from the beginning, middle, and end of the book are then averaged to obtain an estimate of readability for the text.

## Equations

Suppose you work in a food store.
You are to fill a display case with candy.
You have put 16 boxes into the case.
The case can hold 24 boxes.
How many more boxes are needed?

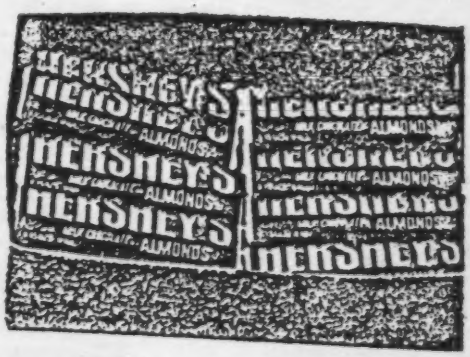
You can write an **equation** for the problem.

Think: 16 plus some number is 24.

Write: $16 + n = 24$

You solve the equation to solve the problem.
Find the number that makes the equation true.
You subtract to find a missing addend.

$$\begin{array}{r} 24 \\ -16 \\ \hline 8 \end{array}$$




8 makes the equation true. $16 + 8 = 24$
8 more boxes of candy are needed.

*No. of difficult words: 4*

*Average sentnece length: 102/15≈6.8*
*≈7 words/sentence*

*Level of difficulty: (4+7) x .4 = 4.4*

*From Growth in Mathematics, Grade 6 by David W. Wells, et al. New York: Harcourt, Brace & Jovanovich, 1978, p. 22.*

The Fry Graph. This technique also does not involve students in the assessment process. It, too, estimates readability by the length of words and sentences. What follows are instructions in the use of the Fry Graph[2].

1. Select three 100-word passages from near the beginning, middle, and end of the book. Skip all proper nouns.
2. Count the total number of sentences in each 100-word passage (estimating to the nearest tenth of a sentence). Average these three numbers (add together and divide by 3).
3. Count the total number of syllables in each 100-word sample. There is a syllable for each vowel sound; for example: cat (1), blackbird (2), continental (4). Do not be fooled by word size, for example: ready (2), stopped (2), bottle (2). It may be more convenient to count every syllable over one in each word and add 100. Average the total number of syllables for the three samples.
4. Plot on the graph the average number of sentences per 100 words and the average number of syllables per 100 words. Most plot points fall near the heavy curved line. Perpendicular lines mark off approximate grade-level areas.

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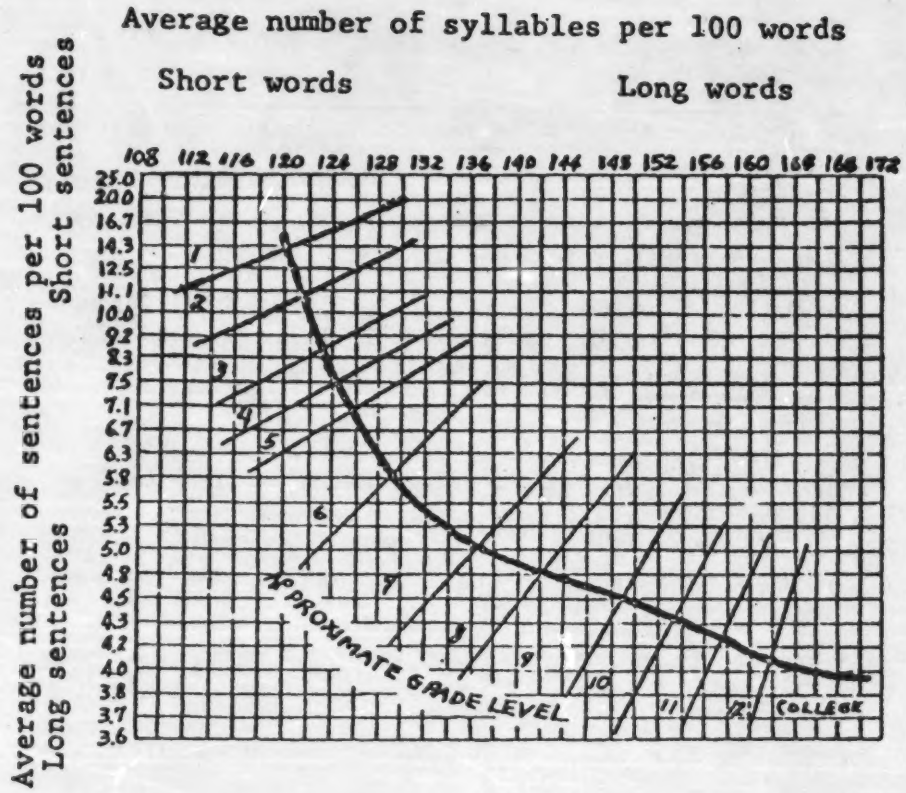
[2]E. Fry. "Readability Formula That Saves Time." Journal of Reading. April 1968, pp. 513-516, 575-578.

THE FRY GRAPH




Whatever assessment technique is used to determine readability of materials it must be borne in mind that it is but one means by which a teacher attempts to obtain a "good fit" between the author's assumptions regarding the reader's language, experience, and thinking background and the reader's actual language, experience, and thinking background. The closer the "fit" is between the author's and reader's assumptions, the easier the reading materials will be for the students.

## Annotated Bibliography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*Gives many practical suggestions (by way of examples) for teaching the reading of mathematical materials.*

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*Discusses the teaching of reading versus teaching of reading in mathematics; symbol perception and vocabulary; and study guides--their nature and construction. Examples of study guides provided.*

Henney, M. "Improving Mathematical Verbal Problem-Solving Ability Through Reading Instruction." Arithmetic Teacher. April 1971, pp. 223-229.
*Discusses ways in which the teacher may help students, especially fourth graders, develop their ability to read verbal problems.*

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*Identifies and discusses the skills of reading as they apply to mathematics in high school.*

Wilson, J. "What Skills Build Problem Solving Power?" The Instructor. February 1967, pp. 79-80.
*There are at least three types of meanings of arithmetic operations: mathematically pure, physical action, and socially significant. Of the three, definitions of operations having socially significant meanings lead us to "wanted-given" relationships. Guiding children to these "wanted-given" relationships has been found to be the most effective major approach to improving their problem-solving abilities. This major approach should be supplemented with cognitive skills such as the use of equations to express "wanted-given" relationships, estimating correct answers, and having children make up their own problems. Examples are given for the elementary level.*